In presenting the dissertation as a partial fulfillment of the requirements for an advanced degree from the Georgia Institute of Technology, I agree that the Library of the Institute shall make it available for inspection and circulation in accordance with its regulations governing materials of this type. I agree that permission to copy from, or to publish from, this dissertation may be granted by the professor under whose direction it was written, or, in his absence, by the Dean of the Graduate Division when such copying or publication is solely for scholarly purposes and does not involve potential financial gain. It is understood that any copying from, or publication of, this dissertation which involves potential financial gain will not be allowed without written permission.

7/25/68

DETERMINATION OF THE MARGINAL GROWTH RATE TO BE USED IN THE MPV CRITERION FOR SELECTION OF INVESTMENTS

A THESIS

Presented to

The Faculty of the Division of Graduate

Studies and Research

by

Alberto Sabino Parra

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Industrial Engineering

Georgia Institute of Technology

June, 1972

DETERMINATION OF THE MARGINAL GROWTH RATE TO BE USED

IN THE MPV CRITERION FOR SELECTION OF INVESTMENTS

Approved by:

Dr. G. J. Thuesen, Chairman

Dr. D. E. Fyffe

Dr. J. Banks

Date approved by Chairman: 8/11/72

ACKNOWLEDGMENTS

The author expresses his deepest appreciation and gratitude to Dr. G. J. Thuesen whose guidance, effort, and inspiration made this work possible; his effort made the completion of this thesis a reality.

In addition, I would like to thank Dr. D. E. Fyffe and Dr. J. Banks for their helpful comments and suggestions concerning all aspects of this research.

The financial support for my studies was provided by Instituto Tecnológio de Monterrey. Without this assistance, such a study would have been impossible.

Acknowledgments are due to my friends for their help and encouragement, especially to Mr. Frank Alt, Dr. Benito Flores, Dr. and Mrs. Gonzalo Mitre, Ing. Francisco Vera, and Mr. and Mrs. Juan Zamarrón.

I would also like to express my gratitude to my parents, Graciela and Sabino, and my grandmother, Dolores Garcia. Special gratitude is due my wife, Adela, for her continued encouragement, love, and assistance.

Finally, I wish to thank Mrs. Betty Sims for her excellent typing job.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

The Capital Rationing Decision

A business firm can be regarded as an entity possessing some resources of money, properties and manpower, that is dedicated to the procurement of the maximum possible benefit for its owners. Investment proposals will be available to the firm which can be regarded as opportunities to exchange some of the resources of the firm for promises of return of some other resources at later dates. In most cases, the total sum of the resources required by the investment proposals will be greater than the resources available for investment. Some organism inside the firm must decide on the allocation of resources to those investment opportunities that, as a whole, return the maximum economic benefits to the firm. In this study, the *capital rationing decision* will be considered as the allocation of limited funds among competing proposals. The organism of the firm in charge of the capital rationing decision will be referred to as Decision-maker.

Importance of the Capital Rationing Decision

It is apparent that the benefits for the firm's owners will be greater if the resources under the firm's control are maximized. The growth of the firm's resources is accomplished through the adoption of proposals that render revenues in larger amounts than the amounts of resources used in their implementation.

The resources available for investment to the firm at a certain moment will determine the firm's ability to undertake some advantageous proposals. If a proposal is undertaken, the firm commits some resources in exchange for revenues at future dates; the amounts and timing of the investments and revenues will affect the amounts of resources available for investment to the firm for some time. The alteration of the resources available in the future will affect the ability of the firm for implementing future proposals, which in turn will affect the growth of the firm's total resources.

The decision maker must determine then whether or not the firm's resources available for investment must be committed in some proposal in exchange of future resources. Successive decisions concerning all of the proposals available to the firm will determine if the firm's resources will grow or diminish and at what rate that change takes place.

Difficulties in Making the Capital Rationing Decision

Many factors must be taken into consideration in order to make the capital rationing decision. For each investment proposal an assessment must be made of the possible benefits or losses, through the prediction of the amount and timing of each investment and revenue to be obtained if the proposal is accepted.

The judgement in the desirability of any proposal in particular must be influenced by the prospective abundance or scarcity of some other attractive investment opportunities in the future. If too many resources are tied up at a given time in low profit, long term investments, the firm may be unable to undertake proposals that may render greater benefits in a shorter time, if they are to be found in the near future. On the other hand, if the decision maker becomes too stringent about the amounts and timing that a proposal's profits must have, the firm may end up with too many resources available, inactive, and too few implemented proposals, even though the benefits to be derived from each of them are relatively high. Either of the two extremes results in a slow growth of the total resources of the firm.

If the capital rationing decision must be made simultaneously for several proposals, the relationships among them must be taken into consideration. Since dependencies or incompatibilities may exist from a technical or financial point of view, their relationships may preclude the simultaneous adoption of some subsets of the set of available proposals. In addition, the availability of proposals at times to come is somewhat uncertain, and the predictions about the amounts and timing of the investments or revenues to be derived from the proposals have varying degrees of accuracy.

In short, the decision maker must decide whether or not a certain proposal must be adopted for implementation by the firm. The decision is made with incomplete information, possibly taking into account some complicated relationships among the proposals, and giving some weight to an opportunity cost for the possible proposals that can be lost because of the unavailability of the resources tied up in the proposal if it is accepted.

Capital Rationing Techniques

Several techniques designed to take into account uncertain future events may assist in determining which investment proposals must be undertaken by the firm. In this study they will be referred to as Capital Rationing Techniques.

The capital rationing techniques vary widely in their nature and complexity. The models of real world situations for which they were derived differ in their assumptions in:

1. the ability of the decision maker to affect the amounts of money budgeted for capital expenditure,
2. the amount of knowledge about present and future investment opportunities,
3. the possibility of accepting fractions of proposals,
4. the possibilities of reinvestment for cash receipts,
5. the existence of relationships among the proposals,
6. whether or not investment opportunities are batched for the capital rationing decision to be made at specified intervals in time,
7. the availability and cost of capital,
8. the objectives of the firm.

In subsequent discussions, any capital rationing techniques that describe a *complete procedure* for determining whether or not an investment proposal or set of investment proposals should be implemented by the firm will be named a *decision criterion* or simply a *criterion.*

The MPV Criterion

In this study the interest is centered in "The Maximum Prospective Value Criterion," developed in 1967 by R. V. Oakford and G. J. Thuesen (1). In subsequent discussions it will be referred to as the MPV criterion. This criterion was developed for a situation where decisions are made sequentially at regular intervals in time with the decision maker bearing no influence over the amount of money budgeted, but having perfect knowledge of present investment proposals and probabilistic knowledge of future proposals. The objective of the firm is stated to be the maximization of its total worth at a future date. The MPV criterion uses a measure of worth for an investment proposal that incorporates a discount rate m for future cash flows, and a rate of interest "i_δ" to be earned if money is invested in some highly liquid fund for one or more periods.

Later in developing the MPV criterion in his doctoral dissertation (2), Dr. Thuesen made a comparison of the effectiveness of the MPV criterion with four other of the most widely known criteria.[1] The method used for the comparison was a simulation.[2] From the results reported, it is apparent that the MPV criterion is superior to the four other criteria as a decision technique. It was found that the discounting rate m presents an optimum value $\bar{m}^*$ that maximizes the growth of a firm's

---

[1]The four criteria used for comparison were (in the notation of Dr. Thuesen): (a) Rank on Growth Rate Criterion, (b) Hunt Criterion, (c) Modified Marginal Growth Criterion, (d) Solomon Criterion.

[2]The results of this later study were also published in (3).

capital under stated conditions. An idealized picture representing the total dollar value of the enterprise at a horizon year H, against different values of m, is drawn in Figure 1.

In the derivation of the MPV criterion (1, page 158), it is stated that $\bar{m}^*$ is the "average marginal growth rate" for the decision maker. The average "marginal growth rate" is the average growth rate at which the cash flows representing the difference between the best decision[1] for a period and the next best decision are invested. Any factor that may alter these marginal differences may have an influence on the value of $\bar{m}^*$.

The range of possible values for $\bar{m}^*$ varies from a low of i_δ, the interest earned on highly liquid investments to a high of $\bar{g}$, the average growth rate of the total capital invested.

The determination of the theoretical value of $\bar{m}^*$ would involve the determination of:

1. the best and the next best decision,
2. the differences among the cash flows that would be generated by the best decision and those cash flows that would be generated by the next best decision,
3. the average growth rate of which the differential cash flows would be invested.

In practice the accomplishment of the process just described would present a great difficulty, since perfect knowledge is required

---

[1]The decision to undertake the set of proposals that maximizes the net expected value of the enterprise's capital at a horizon year.

in future investment opportunities.

For a practical application of the MPV criterion it is necessary to have a good estimate of the value of $\bar{m}^*$ in order to reach good capital rationing decisions through the application of the MPV criterion. But the estimation of $\bar{m}^*$ must be made through the use of some data known to a businessman at least in an approximate form.

Objective of this Study

It is apparent that the theoretical method for the determination of $\bar{m}^*$ does not conform with any of the methods presented in previous works for the determination of the opportunity cost for money.

This study was undertaken with two main objectives:

1. to study the effect of variation in the values of three parameters in the values of $\bar{m}^*$, trying to find some qualitative results,
2. to obtain an understanding of the effect of using the MPV criterion with different tentative discount rates as a decision technique in a sequential decision process.

The technique used for this study was a computer simulation. An investment situation was modeled in which an enterprise started operations at a period $t = 0$ with a certain amount of capital M_0. At period $t = 0$ and subsequent periods $t = 1,2,\ldots,H$, the decision maker for the enterprise received a certain number of investment proposals and decided about the rationing of available capital with the help of the MPV criterion. Two restrictions were present: (a) the budgets for time periods $t = 1,2,\ldots,H$ should be internally generated, (b) there were no dividend payments.

In order to model a firm in an appropriate form for this type of study, several simplifying assumptions were made. A firm was modeled through nine variables capable of mathematical representation. From these variables, three were selected for studying the effect that different investment situations have on the values of $\bar{m}^*$. The three selected variables were:

1. interest rate earned in highly liquid investments,
2. variability in the number of proposals to be considered each period,
3. cash flow shape composition of the proposals.

The numerical values presented are not claimed to have universal application because the situations studied were very specific, and the simplifications used will limit the validity of the results. But the tendencies found in the effect of the three studied factors on the values of $\bar{m}^*$ are believed to have sufficient generality to be of some help in the determination of an approximate $\bar{m}^*$ for a business enterprise.

Plan for this Study

Chapter II presents a short literature survey about capital rationing techniques; Chapter III presents the complete formulation of the MPV criterion along with the reasons for selecting for special study the three factors previously mentioned. Chapter IV presents the simulation assumptions, the general form of the simulation variables, and a description of the procedures used for the simulation. Chapter V presents the particular situations studied, discussion of results, conclusions and recommendations.

CHAPTER II

REVIEW OF THE LITERATURE

This study, as stated in the first chapter, is concerned with the determination of the appropriate "average marginal growth rate" $\bar{m}^*$ to be used in the application of the MPV criterion as a capital rationing technique.

The only specific references about this theme are found in Oakford and Thuesen's works (1,2,3). In this chapter a review is made of some other capital rationing techniques, with special attention to those that consider the time value of money.

Time Value of Money

There is an almost universal agreement among the authors of economic literature in the fact that money has a time value. In plain words, the time value of money expresses the fact that, normally, people prefer to have one dollar at hand *now*, than the promise to obtain one dollar at some future date.

Most of the modern criteria use a weight for the time value of money, and there are several measures of an investment proposal's worth that introduce this concept. The three measures of a proposal's worth defined below are important for the formulation of the criteria to be reviewed later in this chapter.

Present Worth of a Proposal

R. H. Bernhard in his article "Discount Methods for Expenditure Evaluation--A Clarification of Their Assumptions" (4), defines the Present Worth of a productive investment as:

$$P = Q_0 + \frac{Q_1}{(1+i_1)} + \frac{Q_2}{(1+i_1)(1+i_2)} + \dots + \frac{Q_n}{(1+i_1)(1+i_2)\dots(1+i_n)} \qquad (1)$$

where n is assumed to be the life of the project, and

Q_s = Net incremental return to be gained at the end of period s.

s = 1,2,...,n.

i_s = Rate of interest for borrowing or lending in any quantity during period s.

Generally, it is assumed:

$$i_1 = i_2 = i_3 = \dots = i_n = i$$

and the definition becomes:

$$P = \sum_{s=0}^{s=n} (1+i)^{-s} Q_s \qquad (2)$$

An interesting derivation of the present worth measure of value is presented by Williams and Nassar in (5). Their approach was through two axioms about the behavior of people (they assume the existence of greed and impatience), and three axioms about the conditions necessary for the existence of a complete ordering relation for investments describable through vectors $Q_0, Q_1, Q_2, Q_3, \dots Q_n$. (These axioms state the necessity of the existence of continuity and marginal and temporal consistency for a complete ordering relation for this type of vector).

Solving Rate of Return

The solving Rate of Return (SRR) measure of an investment proposal's worth is defined by Bernhard as the i*, or set of i*'s that reduce the discounted sum of the net incremental return of the proposal to a present value of zero. That is,

$$\text{SRR} = \left\{ i^* \middle| 0 = \sum_{s=0}^{s=n} Q_s (1+i^*)^{-s} \right\} \qquad (3)$$

Future Worth

The last measure of an investment's worth to be presented is the Future Worth, as described by G. J. Thuesen in (3). It is is assured that a proposal j can be described through a series of cash flows $S_{0j}, S_{1j}, \ldots, S_{nj}$, where n is the life of the proposal, the future worth of this proposal is:

$$\text{Future Worth } j = \sum_{t=0}^{t=n} S_{tj} (1+i)^{n-t}, \qquad (4)$$

In this formula the implicit assumption is that the receipts from the proposal are invested at a rate i. It is apparent that Future Worth of proposal j is $(1+i)^n$ times the present worth of proposal j.

Definitions

It is now appropriate to define some terms that deal with properties of *sets* of investment proposals. In accordance with Weingartner

(6), it is said that a *set* of investment proposals is:

a. *Independent:* When the worth of individual investment proposals is not profoundly affected by the acceptance of others.

b. *Mutually Exclusive:* When acceptance of one proposal in that set renders all others in the same set clearly unacceptable or even unthinkable.

c. *Contingent:* When the acceptance of one proposal depends on the acceptance of other proposals.

d. *Compound:* When contingent proposals are combined with the proposals on which they depend, so that the independent proposal and the compound proposal may be treated as mutually exclusive alternatives.

Other necessary terms will be defined in the text as they appear.

Criteria for Capital Rationing

Historically, the oldest criterion is to use no defined procedure at all. A decision on the undertaking of an investment opportunity is based on hunches, rumors, experience, intuition and mood of the decision maker. Most of the personal buying decisions (buying of homes, automobiles, home appliances, etc.) are made this way. And a surprising number of large enterprises conduct their business this way, too [8.4 per cent, in a sample of 48 large enterprises, as reported by D. F. Istvan in "The Economic Evaluation of Capital Expenditures" (7)].

The Payback Period method uses some quantitative estimations and it is described by Smith in (8). The Payback Period for an investment proposal indicates the number of years necessary for the recovery of the first cost of the proposal. When several mutually exclusive proposals are considered, the one having the shortest payback period would be selected as "best" by this particular criterion. The payback period of the best alternative is then compared to a maximum payback period determined by the firm as a policy. In the description no mention is made about budget limitations, nor is there mentioned any kind of relationship among the proposals. It is then quite apparent that cash flows occurring before the payback period of the proposal receive a weight of 1, and those effectuated after the payback period receive a weight of 0. An analysis is given in Smith's book about a "rule of thumb" to decide the value of the maximum payback period to be used.

Joel Dean in his book "Capital Budgeting" (9) presents a criterion that has had considerable influence in the capital budgeting literature. His approach was a simplified version of the economic theory of investment, and the resulting criterion recognized the Time-Value of money. The objective was the maximization of the economic welfare for the owners of the firm. The criterion prescribes the computation of the SRR for each investment proposal and the ranking of these proposals in decreasing order of SRR. The intersection of the resulting schedule with the marginal cost of capital schedule determines a "cut-off" rate (or minimum acceptable rate of return) that can be used afterwards for the selection of proposals. It is assumed that the decision maker will borrow as long as it is economically desirable. Some of the limitations of this criterion are the lack of provision for non-independent proposals, the possibility that the set of SRR has more than one element or none at all, a practice called "preliminary selection," that is encouraged by this criterion. Also it yields poor results in situations with fixed budgets and does not have provisions for the problems posed by non-divisible proposals. The merit of this criterion was that through a wide diffusion, it made the businessmen aware of the necessity to include some sort of valuation for the value of money through time.

In the article "Two Major Issues Associated with the Rate of Return Method for Capital Allocation: The 'Ranking Error' and 'Preliminary Selection'" (10), Gerald Fleischer presents a criterion that deals effectively with non-independence relationships and establishes a method for dealing with limited budgets.

Basically, the criterion (to be named Ranking on Rate of Return of the Incremental Investment Criterion, or RORII for short) calls for the consideration of *all* the possible combinations of proposals that are feasible (taking into account all the relations of dependency, contingency, mutual exclusiveness among the proposals, and the limitations in budget, if present). Each combination is regarded, then, as a mutually exclusive alternative because, in fact, each combination of proposals prescribes an action that is mutually exclusive from the actions prescribed by other combinations. The next step is to rank all these feasible, mutually exclusive alternatives in increasing order with respect to initial investment, and the computation of the SRR over each incremental investment. The increment in investment is undertaken if the SRR for the increment is greater than a minimum attractive rate of return. No mention is made of a method for the selection of the minimum attractive rate of return. This criterion is important because of the considerations made for dealing with non-independent proposals and fixed budgets. These considerations gave as a result a criterion that considers the time value of money and at the same time has general applicability.

The problem presented by the presence of multiple elements in the set of Solving Rates of Return is solved in a rather complicated way in an article of Teichroew, Robichek and Montalbano (11). The authors comment in the conclusions of the cited article that their algorithm gives the same results as the present worth method to be presented next.

The present worth criterion as presented by E. L. Grant and W. G. Ireson in "Principles of Engineering Economy" (12), can be applied to independent and mutually-exclusive proposals. This criterion calls for the ranking of all the alternatives in increasing order of initial investment, the computing of the present worth on an incremental basis at a specified minimum attractive rate of return, and the undertaking of the incremental investment if its present worth is greater than zero. This criterion has been long favored in the economic literature, yet it does not consider explicitly the situations created by contingency relationships among the proposals.

An article that attempts a solution to the problem posed by the presence of disbursements and budget ceilings for two periods was published by Lorie and Savage in "Three Problems in Rationing Capital" (13). In their method they attempt the generation of classes of solutions through an iterative process. This criterion uses the Present

Worth measurement of an investment worth. The procedure prescribes the tentative assignation of different relative weights to money disbursed in different periods of time through a trial-and-error procedure, trying to exhaust all the budget ceilings. There is no indication about when to stop the procedure unless a solution complying with all the budget requirements is found.

This article virtually gave an integer programming formulation for the problem posed by restrictions of fixed capital for one and two periods.

In 1962 Weingartner (6) suggested linear and integer programming formulations for capital budgeting problems. His models include the possibility of borrowing and the delaying of investments. They also consider budget ceilings in several periods, optimization of sources of capital, and include easy formulations for non-independence relationships among the proposals. The objective functions for their formulations are linear combinations of the future worths of the proposals and the money left after the last period's investments are made. His models require complete knowledge concerning present and future proposals. These formulations were significant because of the use of linear and integer programming algorithms which search all the feasible alternative proposals without an explicit enumeration of them. The question about the selection of a discounting or compounding rate is not answered.

In 1967 Oakford and Thuesen (1) presented the formulation of the Maximum Prospective Value criterion (MPV); this criterion is based on an integer programming model which assumes sequential decisions at regular intervals in time, fixed budgets, perfect knowledge about proposals, a probabilistic knowledge about future investment opportunities and some other features.

The objective function is a linear combination of a function called Prospective Value of the Proposal which involves a discounting rate, $\bar{m}$, and a rate of interest i_{δ} that is earned on highly liquid investments. A more complete comment is reserved for the next chapter.

Selection of Rates

Some authors discuss the selection of the appropriate rate to be used in the application of some criterion to a specific investment situation. The nature of these comments varies widely, from rather extensive discussions presented in textbooks to very brief, elusive comments presented in most articles. Quoted from the previously-cited article published by Lorie and Savage, the next paragraphs present the attitude encountered in most of those articles.

> The question of determining the cost of capital is difficult, and we, happily, shall not discuss it. Although there may be disagreement about methods of calculating a firm's cost of capital, there is substantial agreement that the cost of capital is the rate at which a firm should discount future cash flows in order to determine their present value.
>
> . . . .
>
> One of the difficulties with the concept of cost of capital is that in complicated circumstances there may be no one ratio that plays this role. Even worse, the very concept of present value may be obscure.

In some textbooks for the case of no limitation in the borrowing power of the decision maker, it is recommended that the cost of capital as a cutoff rate for the RORII criterion and as a discounting rate for the present worth method.

For the case in which the decision maker has limited capacity for the alteration of the amount budgeted, some of the articles and textbooks provide some help. In Grant and Ireson's widely-known textbook (12), Chapter 9 is devoted to a discussion of the appropriate discount rates to be used. Through the chapter the authors point out the difference of considerations necessary for the models with limited and unlimited funds and cases where borrowing is allowed. Quoting the conclusions of Chapter 9 (page 157), ". . . the major point to keep in mind is that the attractive return should be either the return on the investment opportunity foregone, or the overall cost of capital."

Some sources state guide lines to obtain an approximation of the cost of capital. Rather detailed analysis from a financial point of view can be found in Chapter 19 of (3) and Chapter 9 of (12). An article co-authored by Brigham and Smith (14) particularly emphasizes the determination of cost of capital for small firms as a weighted average of the cost of money provided by different sources.

CHAPTER III

BACKGROUND

This chapter contains definitions of terms, the formulation of the MPV criterion, and an exposition of the reasons that led to the selection of the three factors investigated in this study.

This study will adopt the nomenclature used in the article, "The Maximum Prospective Value Criterion" (1); a summary of notation is presented in Table 1.

The *capital* of the firm will be considered as the total financial commitment of the firm and it will be visualized as growing through time at a rate $\bar{g}$, called the *growth rate* or the *capital growth rate*. The growth of the firm's capital is considered to be due to productive investments.

All the rates used will be referred to as *growth rates*; for example, for a proposal the Solving Rate of Return of proposal "j" (SRRj) will be referred to as the *prospective growth rate of proposal "j"* or g_j. It is assumed that interest is a special case of growth rate. The interest earned on money invested in highly liquid funds will be termed i_δ.

The term *budget*, for a period, will refer to a dollar ceiling for capital investments, fixed by the allocation of funds by the firm.

The time origin for the notation used in the formulation of the MPV criterion will be period 0. The index $t = 0,1,2,\ldots,H$ will be used

Table 1. Summary of Notation*

g	is a generic symbol for a growth (discount) rate to be used in computing future values or present worths.
g_j	is the prospective growth rate that will result from the investment made by the decision maker in the time interval t=1 to T=H.
H	represents the decision makers' horizon time.
i_δ	is the current interest (growth) rate obtainable by the decision maker on highly liquid investments (e.g., bank savings accounts).
j	1,2,3,..., is an index on groups of proposals included in a schedule of investment proposals.
m	is the decision maker's marginal growth rate, assumed to be effective from time t=0 to T=H.
$M_{\delta x}$	$= M_0 - \Sigma X_j S_{j0} \geq 0$ is that part of the money that has been budgeted for investment at time t=0 but not absorbed by the combination X.
M_t	is the amount of money that has been budgeted for investment at the time of decision t.
S_{jt}	is the net cash flow at time t for proposal j. $S_{jt} > 0$ indicates cash received by the decision maker. $S_{jt} < 0$ indicates cash disbursed by the decision maker.
X	is an index on the combinations of proposals that can be formed from the proposals in the decision maker's schedule.
X_j	= 1 if proposal j is included in combination X. Otherwise $X_j = 0$.

*Excerpts from Table IX from (1).

to represent successive periods at integer points through time, where H is defined to be a horizon date for the firm.

The term Schedule of Investment Proposals (SIP) is used to represent the set of Investment Opportunities to be considered in a particular period by the decision maker.

At the instant t=0 and at the end of each successive period, the decision maker will consider the schedule of investment proposals presented to him and will decide what subset of it will be undertaken, with the necessary considerations for non-independence among proposals and budget limitations. The funds left after the investments are made at the beginning of each period will be invested in a highly liquid fund for one period at interest i_δ.

In the formulation of the criterion, an explicit consideration of non-independence relationships is not made.

The assumptions used for the derivation of the MPV criterion are the following (1):

1. A fixed budget is determined by the firm, with the decision maker having no influence on the securing of external capital.

2. The investment proposals are batched, and decisions regarding them are made at regular intervals in time.

3. Given a certain set of investment proposals, and a fixed amount of money, the proposals to be undertaken will be the same, without regard to the origin of that money.

4. The investment proposals can be fully described through a set of cash flows.

5. If an investment proposal is undertaken, the previewed cash flows will occur in the proper time; i.e., the decision maker possesses perfect knowledge about all of the proposals under consideration.

6. There is not an exact knowledge about the proposals to be considered in subsequent periods, but a statistical knowledge about what can be expected is available.

7. The objective of the Maximum Prospective Value Criterion is the maximization at a horizon time H of the future value of the investments recommended by the criterion at the decision time.

8. The difference between the amount budgeted and the amount of money actually invested in any decision period will be invested in highly liquid funds, at an interest i_δ, which is less than the average growth rate of the total worth of the firm.

9. The rejected proposals will be dropped and will not be considered in future decision periods.

Formulation of the MPV Criterion

The formulation of the MPV criterion is as follows (1, pp. 152-153):

Assume that the decision maker knows:

1. the amount of money M_0 budgeted for investment at the decision time $t = 0$,
2. his investment function or at least his marginal growth rate m, and the rate i_δ to be earned in highly liquid investments. These values are assumed to be constant from the time of decision until his horizon time,

3. his schedule of investment proposals, with each investment proposal being represented by a series of cash flows. Let s_{jt} represent a cash flow at time t for proposal j, $s_{jt} < 0$ represent a disbursement, and $s_{jt} > 0$ represent a receipt. In this formulation it is assumed for all proposals j that $s_{j0} < 0$ and $s_{jt} \geq 0$ for $t > 0$; i.e., each proposal requires that its entire investment be made at time 0,

4. his horizon time H.

It is assumed further that the decision maker wants to maximize his net value at his horizon time and that non-monetary considerations are either nil or that the decision maker will weigh them in his judgement.

Given these assumptions, it is recommended that the decision maker do the following:

1. Compute the present worth at growth rate m of the prospective net value of each proposal j

$$P_j(m) = S_{j0}\,\frac{(1+i_\delta)}{(1+m)} + \sum_{t=1}^{H} S_{jt}(1+m)^{-t} \tag{5}$$

2. Select the combination X of proposals that maximizes

$$P_x(m) = \sum_j X_j P_j(m) \tag{6}$$

subject to the restrictions

$$M_{\delta x0} = M_0 + \sum_j X_j S_{j0} \geq 0 \qquad \text{and} \qquad X_j = 0 \text{ or } 1 \tag{7}$$

The Best and Next Best Solutions

Suppose that a feasible solution X is obtained for the Integer Programming formulation of the MPV criterion, and furthermore, this solution X is adopted as a basis for decision, i.e. any proposal "j" is adopted for investment if $x_j = 1$ and rejected if $X_j = 0$.

From the assumption of perfect knowledge about the cash flows of the proposals, the cash flows at time t, derived from a solution X, will be:

$$S_{Xt} = \sum_j X_j S_{jt} \tag{8}$$

For the MPV criterion, the best decision is defined to be the decision to undertake for investment the set of proposals which maximizes the Prospective Value:

$$P_x(m) = \sum_j X_j P_j(m) \tag{9}$$

Suppose that such a solution XB is found, and the next best solution XNB is determined, too.

There will be a difference in the cash flows determined by these two solutions; the differential cash flows will be:

$$S_{Dt} = \sum_j (XB_j - XNB_j) S_{tj} = S_{XBt} - S_{XNBt} \tag{10}$$

In the economic engineering literature, it is considered that the differential cash flows S_{Dt} are invested in proposals with *marginal growth rates*.

The *average marginal growth rate* at which the differential flows previously described are invested is defined to be $\bar{m}^*$ (1, p. 158).

From this point of view, it is obvious that any variation that may alter the size of the differential flows, or the growth rates of the proposals in which these differential flows are invested, will alter the values of $\bar{m}^*$.

Effect of the Discount Rate m

From another point of view, it is necessary to discuss the influence that the discount rate m used in the MPV criterion has in the decisions to undertake a certain set of proposals.

Let us visualize the decision to undertake a certain investment proposal as the act of exchanging some funds in a present date in exchange of some funds to be received later. It is possible that such a proposal may produce very small revenues during its first periods of existence, bringing some greater quantities several periods later. So, the decision of undertaking such a proposal will "tie up" some funds, precluding the opportunity to invest in some better proposals that may appear in a subsequent period. Thus, an opportunity price is given to the money. In the MPV criterion, this opportunity price is designed as m. If a decision maker guesses an excessively high opportunity price m, small Prospective Values $P_j(m)$ are determined for the proposals in the SIP, and too much money is invested in highly liquid funds earning an interest of i_δ to "wait" for better opportunities expected to come at some later period. The result is a low growth rate for the firm's capital. In the other extreme, a value of m too low may determine high Prospective Values $P_j(m)$ for the proposals in the SIP and an excessive amount of money will be "tied up" in investment proposals with low revenues, losing the opportunity to invest in better proposals in subsequent periods, which will result in a low capital growth rate.

An idealized picture is drawn in Figure 1 to explain the visualized relationship of opportunity cost m and capital at a horizon year.

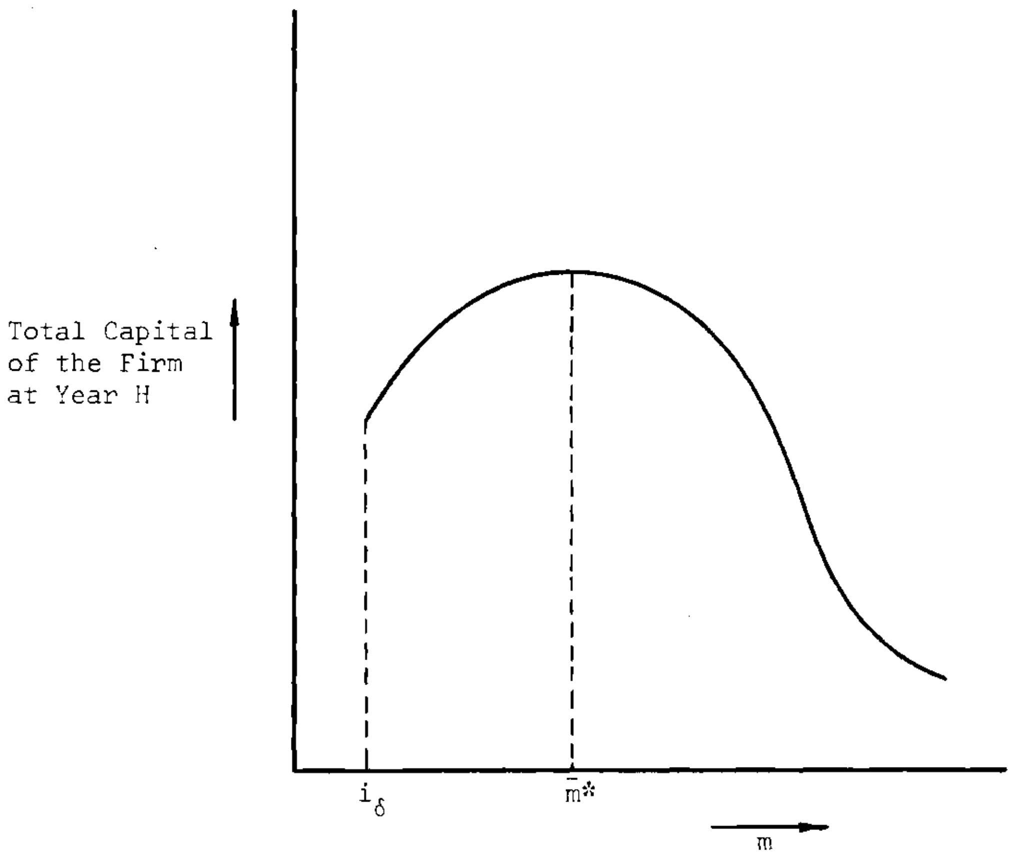


Figure 1. Relationship Among Capital at a Horizon Date and m

The performance of the MPV criterion as a decision technique is optimized for a certain rate $\bar{m}^*$, which can range from i_δ to the growth rate of the invested capital g.

Value of $\bar{m}^*$

A number of factors affect the value of $\bar{m}^*$. The results presented by G. J. Thuesen in (2) suggest an influence from the number of proposals available, the average fraction of the budget that the first cost of these proposals represent, and the variability in the number of proposals to be considered each period.

In fact, a large number of factors that may affect the value of $\bar{m}^*$ are interrelated and difficult to isolate. Three of them selected for this study are:

1. interest earned in highly liquid investments i_δ,
2. variability in the number of proposals to be considered each period,
3. cash flow pattern composition.

The reasons for choosing these factors are:

1. Some money from the differential cash flows determined by X_B and X_{NB}, will be invested in highly liquid funds, earning an interest of i_δ for one or more periods. From this situation, the average growth rate at which the marginal cash flows are invested will be higher with higher values if i_δ. This, in turn, will render higher values of $\bar{m}^*$.

2. An increase in the variability in the number of proposals would affect the values of $\bar{m}^*$, increasing the marginal difference in some periods and reducing it in others. The overall expected effect

was some increase in the values of $\bar{m}^*$ with increased variability.

3. By cash flow pattern, it is understood the geometrical representation of the particular vector that describes the amount and timing of the cash flows for a proposal. For example, if an investment proposal is received in which it is required to invest ten dollars now in exchange for four payments of three dollars each in the following four periods, the geometrical representation would be as presented in Figure 2.

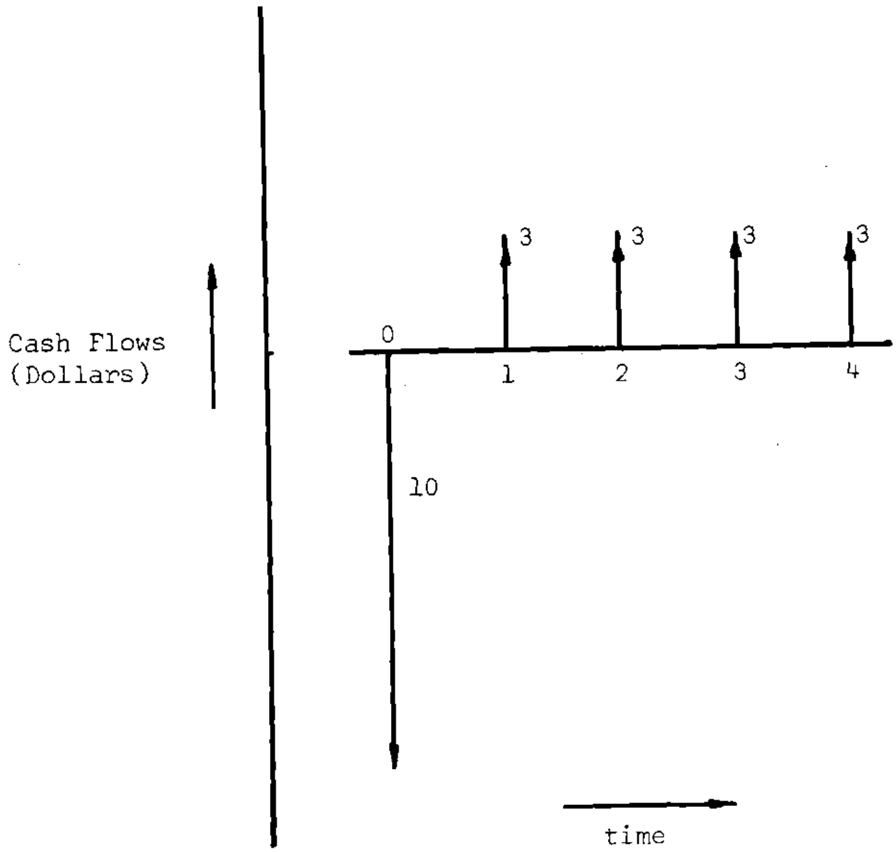


Figure 2. Geometrical Representation of an Investment Proposal

It was pointed out previously how the amount and timing of the cash flows of the proposals accepted in a particular period affect the amount of money available for the firm at future dates. It is reasonable to expect that a firm that receives proposals which generate few cash flow receipts, but in large lumps, may have high variations in the amounts of money available in each period. These variations in turn will originate a broader range for the marginal growth rates and higher values for $\bar{m}^*$ than those appropriate for a firm obtaining proposals with more regular cash flows.

The four cash flow patterns investigated were named:

a. Single Payment.

b. Uniform Series.

c. Increasing Series.

d. Decreasing Series.

These basic cash flow patterns are presented in Figures 3, 4, 5 and 6.

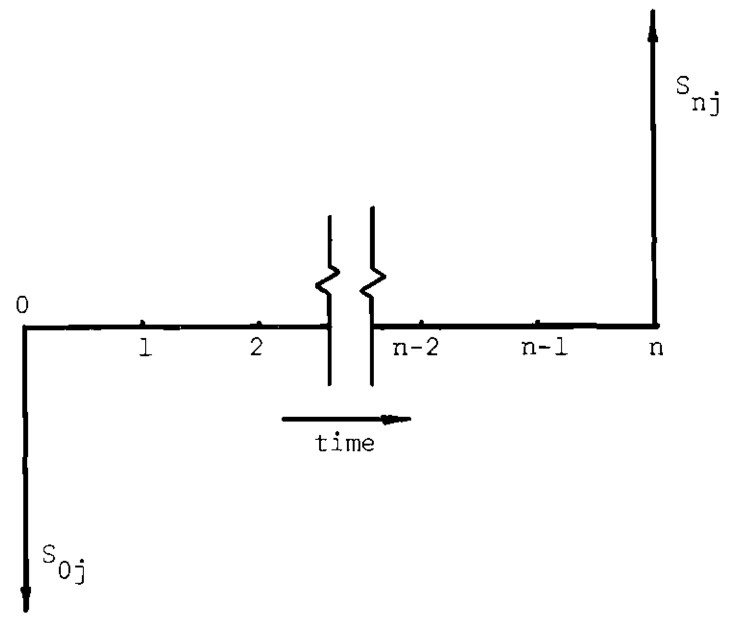


Figure 3. Single Payment Cash Flow Pattern

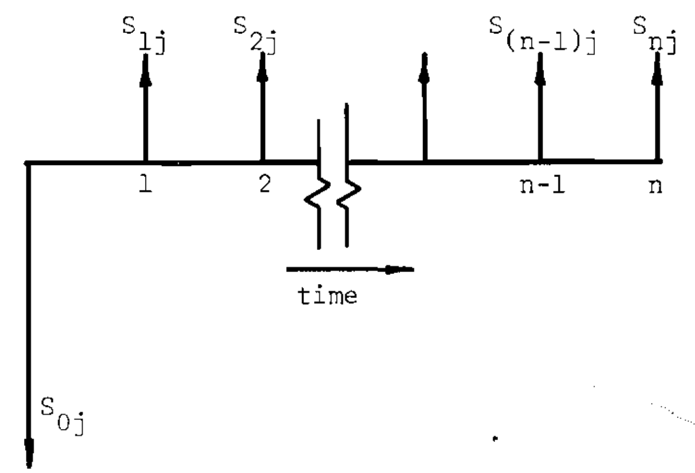


Figure 4. Uniform Series Cash Flow Pattern

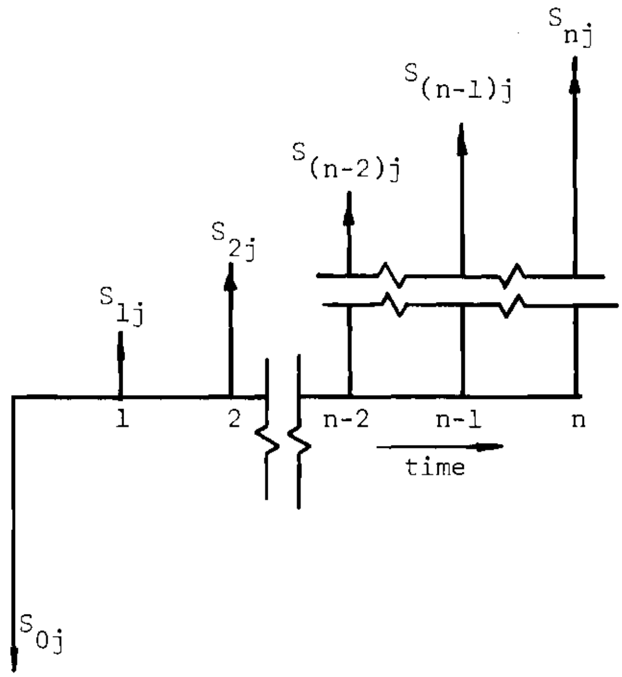


Figure 5. Increasing Series Cash Flow Pattern

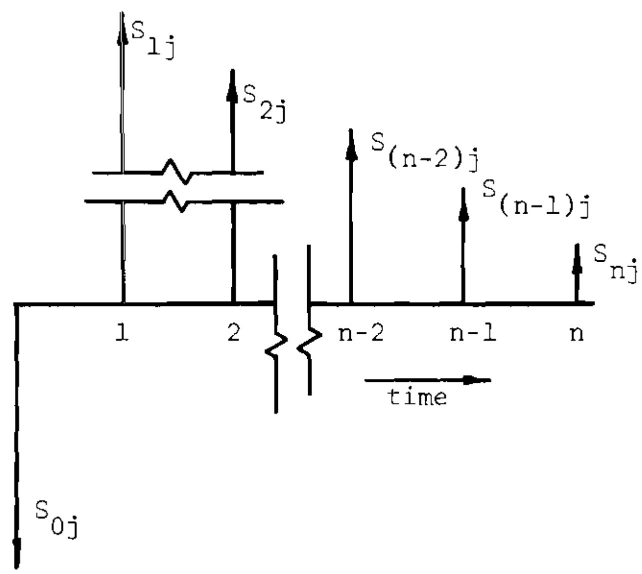


Figure 6. Decreasing Series Cash Flow Pattern

CHAPTER IV

THE COMPUTER SIMULATION

This chapter contains a description of the computer simulation of a sequential decision process. The first part describes the idealized situation envisioned; subsequent parts present the mathematical description of the simulation model, the definition of measures of effectiveness and the procedures used.

Verbal Description of the Decision Process Under Study

In this section a verbal model is presented describing an idealized firm and its decision process. This model is used afterwards for the construction of a mathematical model suitable for computer simulation.

The Firm

In this study a firm will be envisioned as an organism with some monetary resources under its control. No consideration is made of any type of limitations to the operation of the firm apart from financial restrictions. The capital of the firm is considered to be its total financial commitment, and is envisioned as growing through time due to productive investment of money. The objective of the firm is the maximization of its capital at a horizon date, H.

The Investment Proposals

Through the normal operation of the firm a number of investment opportunities are developed at different times. These proposals are collected and considered for adoption by the firm at the end of equal, successive intervals in time; the set of proposals collected for consideration at the end of any period "t" will be named Schedule of Investment Proposals for Time "t" (SIP(t)).

Each investment proposal "j" is assumed to be completely describable through a vector of cash flows, $S_{0j}, S_{1j}, \ldots, S_{nj}$, where n is the life of the proposal. If a proposal "j" is undertaken, it is supposed that the cash flow S_{0j} is accomplished immediately after the decision is made, and each of the subsequent cash flows $S_{1j}, S_{2j}, \ldots, S_{nj}$, is accomplished at the end of periods 1,2,...,n of the life of the proposal, respectively. Furthermore, the cash flows $S_{0j}, S_{1j}, \ldots, S_{nj}$ comply with the following conditions:

$$S_{0j} < 0 \tag{11}$$

$$S_{tj} \geq 0 \quad \text{for} \quad 1 \leq t \leq n - 1$$

$$S_{nj} > 0$$

$$\sum_{t=0}^{t=n} S_{tj} \geq 0$$

In their article *Mathematical Analysis of Rates of Return Under Certainty*, Teichroew et al. (11) show that this condition insures the existence of one and only one growth rate $g_j \geq 0$. This structure of cash flows is reported to be the one most commonly encountered in the normal business operation of most industrial enterprises (6).

Each schedule of investment proposals for a period "t", SIP(t), is made of technically independent proposals, the only limitation being budgetary.

If during the operation of the firm a proposal "j" is undertaken, the forecasted flows $S_{0j}, S_{1j}, \ldots, S_{nj}$ will occur with the predicted timing. In other words, the predictions regarding the proposals' cash flows are perfect. In a real world firm with non-perfect but unbiased predictions and a large number of proposals to be considered each period, the effect of this limitation may be small because of an averaging effect of the errors in the forecasted cash flows of the undertaken proposals.

<u>The Decision Maker</u>

This is a function within the firm whose purpose is to decide at integer points in time $t = 0,1,\ldots,H$ what subset of the set of investment proposals for time "t" shall be undertaken through the application of the MPV criterion. The decision maker is not allowed to decide on the external procurement of funds, i.e., he operates under a budget ceiling.

Three Policies of the Firm

1. For each decision period, *all* the investment proposals generated by the firm are presented to the decision maker. A basic condition for the use of the MPV criterion is that the firm has a policy of centralized decision making, expressly disallowing any "screening" process.

2. Dividends payment is not made throughout the operation of the firm from $t = 0$ to $t = H$. This policy is adopted to avoid a dampening effect in the growth of the firm and the effect that a particular dividend payment policy would have on the growth rate of the firm.

3. New investments are financed through internally generated funds; the amount budgeted for capital expenditures in a given period "t", $t > 1$, will be composed of receipts due at time "t" from previously undertaken proposals plus money left from the previous decision period and invested in a highly liquid fund. In Weingartner's work this investment policy is reported to be commonly employed by industrial enterprises.

The Decision Process

The firm starts operations at period $t = 0$ with a capital M_0 in cash, and this is the only injection of external capital.

At period zero the set of investment proposals, SIP(0), is presented to the decision maker and he decides which subset of SIP(0) shall be undertaken through the application of the MPV criterion and under the restriction of the budget M_0. The remaining money will be invested for one period in a highly liquid fund at interest i_δ.

At each subsequent decision period, $t \geq 1$, the decision maker is presented the SIP(t) and decides through the application of the MPV criterion what subset of SIP(t) will be undertaken under the restrictions of a budget composed of $(1+i_\delta)$ times the money left out from the budget at period $t - 1$, plus any receipt due at time t from investment proposals undertaken previously.

The decision maker knows the interest rate, i_δ, and a tentative marginal growth rate, m. Both i_δ and m are assumed to constant through time, from $t = 0$ to $t = H$.

Definition of Variables

In a real world situation the different factors that affect a firm are closely interwoven and are not easily identifiable. For this study some simplifying assumptions were necessary to define some factors as separate entities, and some relationships must be established among these factors.

For this work some simplification is already made under the verbal model. Further assumptions presented later in this section will permit a mathematically modeled sequential decision process; these assumptions will limit the real world applicability of the numerical values for $\bar{m}^*$ extracted from the model, but it is believed that the tendencies found in the influence of some factors over the values of $\bar{m}^*$ are applicable to real world firms.

Mathematical Description of the Simulation Model

The verbal model will now be translated to a mathematical model.

In order to take advantage of the availability of Fortran IV programs for the generation and analysis of data for the simulation of a firm constructed by Dr. Thuesen (2), the definition of the factors was made in such a form that the values of all variables and the forms and values for all the probability distributions used could be injected as data in the generation program.

Investment Situations

A firm has innumerable characteristics that may define it. For the mathematical model used, the following factors were selected as parameters:

1. an interest rate earned on highly liquid funds, i_δ,
2. a probability distribution for the cash flow shape of the proposals of SIP(t),
3. a probability distribution for the number of proposals contained in SIP(t),
4. an initial capital, M_0,
5. a horizon date H,
6. a probability distribution for the growth rate g_j of the proposals of SIP(t),
7. a probability distribution related to the life of the proposals contained in SIP(t),
8. a probability distribution for the first cost of the proposals contained in SIP(t).

An *investment situation* will be determined by particular values for M_0, H, and i_δ, and particular probability distributions for the other five factors.

Description of the Variables

The form of the variables that describes an investment situation is now presented. Some of these variables were kept constant as parameters through the whole study.

Interest Rate on Highly Liquid Funds i_δ

This rate is expressed as a percentage. It is one of our variables under study. In order to observe its effect, two values of i_δ were studied: 4 per cent and 10 per cent. Most of the work was done at 4 per cent.

Cash Flow Shape Distribution

Cash flow shape of a proposal "j", CFj will be understood to be the particular configuration of the cash flows of such a proposal (see Figures 2, 3, 4, 5 and 6).

From the conditions stated in the verbal description of a proposal (p. 38) it is known that any proposal "j" under our consideration must have a unique growth rate g_j.

The number of vectors S with initial cost S_{0j}, life $L_j > 1$, and growth rate g_j, is infinite since by definition of g_j

$$0 = \sum_{t=0}^{M} S_{tj}(1+g_j)^{-t} \tag{12}$$

Each set of S_{tj} that satisfies this equality with values

$$S_{tj} \geq 0 \quad \text{for} \quad 1 \leq t \leq n-1, \quad S_{nj} > 0 \tag{13}$$

would be one such a vector. In order to quantify this factor, four cash flows are defined as basic: a vector $S_{0j}, S_{1j}, \ldots, S_{nj}$ is said to have a cash flow pattern of the type:

1. *Single Payment* (SP) if the cash flow pattern has:

$$S_{0j} < 0, \quad S_{tj} = 0 \quad \text{for} \quad 1 \leq t \leq n-1, \quad S_{nj} > 0 \tag{14}$$

2. *Uniform Series* (US) if the cash flow pattern has:

$$S_{0j} < 0, \quad S_{tj} = S_{uj} \quad \text{for} \quad t,u = 1,2,\ldots,n \tag{15}$$

3. *Decreasing Series* (DS) if the cash flow pattern has:

$$S_{0j} < 0, \quad S_{tj} = S(n+1-t) \quad \text{for} \quad 1 \leq t \leq n, \quad S > 0 \tag{16}$$

4. *Increasing Series* (IS) if the cash flow pattern has:

$$S_{0j} < 0, \quad S_{tj} = tS \quad \text{for} \quad 1 \leq t \leq n, \quad S > 0 \tag{17}$$

Any proposal "j" describable through a cash flow vector $S_{0j}, S_{1j}, \ldots, S_{nj}$, and a solving rate of return g_j can be separated into a set of basic cash flows. The procedure for this breakdown can be found in almost any book of Engineering Economy.

In order to isolate and study the effect of the presence of different proportions of basic cash flow patterns among the investment proposals considered by the firm, it is assumed that each investment proposal conforms exactly to one of the four basic cash flow patterns. The cash flow shape of a proposal is a random variable, and the probability distribution associated with the cash flow shapes is of a multinomial type with:

$$P_{sp},\ P_{us},\ P_{is},\ P_{ds} \geq 0 \tag{18}$$

$$P_{sp} + P_{us} + P_{is} + P_{ds} = 1 \tag{19}$$

where P_{sp} = probability of SP cash flow pattern.

P_{us} = probability of US cash flow pattern.

P_{is} = probability of IS cash flow pattern.

P_{ds} = probability of DS cash flow pattern.

In a firm in which there are several cash flow patterns, i.e., P_{sp}, P_{us}, P_{is} and $P_{ds} > 0$ with some specific values, the adoption of several proposals with different cash flow patterns may resemble the stream of returns for a real world firm.

Probability Distribution for the Number of Proposals to be Considered Each Period

For this distribution the form adopted was a normal distribution truncated at the left. The mean was set at a value of 10, and the standard deviation was set to a value of 0 for most of the study. In some comparative runs made in order to observe the effect of variability, the standard deviation was set at values of 2 and 4. The distribution was truncated at the left since the minimum number of proposals to be considered in a given period was set to a value of 1.

Initial Capital M_0

This factor represents the amount of money available at the moment in which the firm starts operations; that is, it is the capital of the firm at t = 0. Four levels of M_0 were studied: 10^5, 2×10^5, 3×10^5, 4×10^5. For a better comprehension the values of this factor can be visualized as dollars. Furthermore, the data presented may be understood more easily if the periods are visualized as years.

Horizon Date H

This is the last decision period to be considered for a firm. If H were too small, the results should have a large variability, too large a value of H would require too much computation time. The value used for all the simulations in the computer work was H = 14. This value of H proved satisfactory for similar situations in the investigation reported by G. J. Thuesen (2).

Probability Distribution for Growth Rate of Proposals

The existence of a probabilistic distribution f(g) is assumed. It has the form: Probability of an investment Proposal "j" having a growth rate less than "g_j", where

$$"g_j" = F(g_j) = \int_{i_\delta}^{g_j} f(g)dg \tag{20}$$

This distribution has a lower limit higher than i_δ because it is assumed that the firm can *always* lend any quantity of money at interest i_δ for one period. The "curve of investment opportunities" of the economists can be visualized as the graph of $F(g_j)$ rotated 90° counter-clockwise. This distribution was kept constant through the study and is presented in Table 2 and in Figure 7.

Table 2. Probability of a Proposal j of Having a Growth Rate of g or Greater

g	Probability of gj ≥ g	g	Probability of gj ≥ g
.30	.014	.17	.269
.29	.028	.16	.298
.28	.043	.15	.330
.27	.059	.14	.366
.26	.075	.13	.405
.25	.092	.12	.446
.24	.110	.11	.5
.23	.129	.10	.550
.22	.149	.09	.629
.21	.170	.08	.716
.20	.192	.07	.830
.19	.216	.06	1.0
.18	.241		

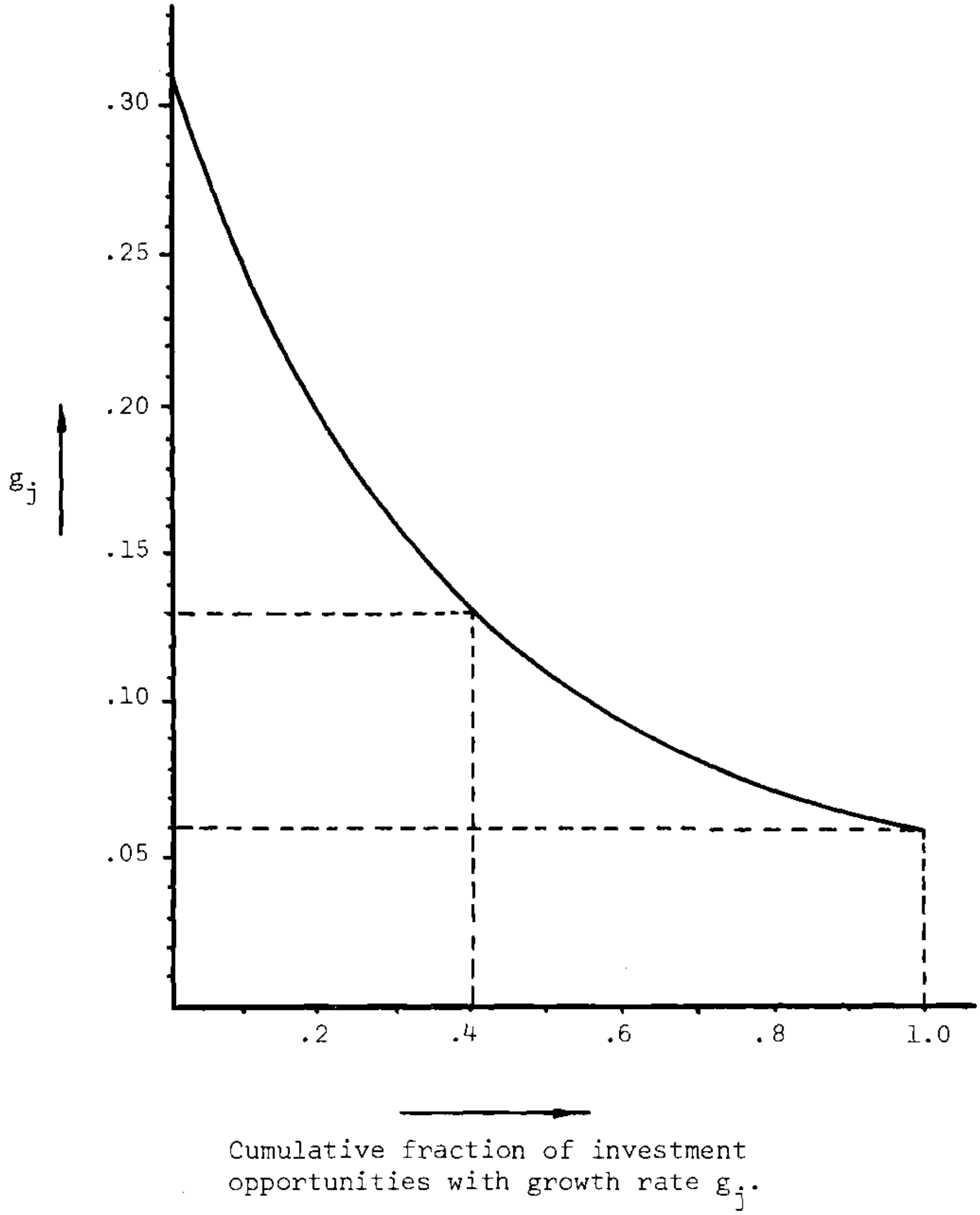


Figure 7. Curve of Investment Opportunities

The maximum value for any g_j is 30 per cent, and the lower limit is 6 per cent. The definition of this distribution was made in a purely subjective form. The shape of this distribution gives higher probabilities to the availability of proposals with low growth rates, and small probabilities to proposals of high growth rates--a situation which is commonly found in real world situations.

Probability Distribution of the Life of the Proposals

The life of the investment proposals "L" is considered to have a probability distribution f(L). This distribution has the form:

Probability of

$$(L<Lj) = F(Lj) = \int_a^{Lj} f(L)d_L \tag{21}$$

where a is a lower limit, $a \geq 0$.

This probability distribution was considered to remain constant through the life of the firm.

The particular form adopted for this distribution was the general exponential distribution

$$f(x) = \begin{cases} 0 & , \quad x<a \\ \lambda \exp\left[-\lambda(x-a)\right], & x\geq a, \quad \lambda>0 \end{cases} \tag{22}$$

This distribution was selected because it describes a common situation where short-life proposals are more frequent than long-life proposals.

The particular values of a and λ used were $a = 2$ and $\lambda = 3$ through the whole study, and this gives an average of five periods for the life of the proposals.

Probability Distribution for the First Cost of Proposals

It is logical to expect that for a growing firm the sum of the first costs of the proposals contained in the SIP(t)'s for $t = 0,1,2,\ldots,H$ will be increasing at some rate. This increase will be due to augments in the average number of proposals per period, and some growth in the average first cost of the proposals. This study has an interest in the effect of variability of the number of proposals, and because of this, it was decided to preserve unaltered the probability distribution that describes the number of proposals per period through the operation of the firm and to consider the average first cost of the proposals as growing through time at a rate gp in order to compensate for the growth of the budgets.

It is easy to visualize that without this provision (i.e., if the average cost of the investment proposals and the distribution related to the number of proposals per period are kept constant) after some periods of operation the budget would be high enough to undertake all the proposals contained in the SIP(t)'s with Prospective Values greater than zero. This situation does not bear interest within this work.

In order to facilitate the computer procedures, a static distribution for the first cost of proposals is an input to the simulation and preserved as constant through the existence of the firm, $t = 0,\ldots,H$.

The first cost of a proposal "j" at time "t" is obtained through one random sampling in the static distribution which gives a value S^*_{0j}. The first cost of the proposal "j" to be considered at time t is determined as:

$$S_{0j} = S^*_{0j}(1+gp)^t \tag{23}$$

The static probability distribution is constructed by the combination of three general exponential distributions.

The three general exponential distributions have the form:

$$f_i(x) = \begin{cases} \lambda_i \exp\left(-\lambda_i(x-a_i)\right), & x \geq a_i, \quad \lambda_i \geq 0 \\ 0 & , \quad x < a_i, \end{cases} \tag{24}$$

$$\text{for } i=1,2,3, \; a_i \geq 0, \; \lambda_i > 0, \; a_1 > a_2 > a_3$$

The expected value of a variable x_i related with any one of these distributions is:

$$E(X_i) = a_i + \lambda_i = C_i, \quad 1=1,2,3. \tag{25}$$

The data fed into the program are: the average first cost expected from all the proposals, C_0, and the value- of a_1, C_1, a_2, C_2, a_3, C_3.

It is required that $C_2 \leq C_0 \leq C_3$.

The computer samples from the three distributions with probabilities f_1, f_2, f_3 which satisfy the following restrictions:

$$f_1 + f_2 + f_3 = 1 \tag{26}$$

$$C_1f_1 + C_2f_2 + C_3f_3 = C_0$$

$$C_1f_1 - C_2f_2 = 0$$

$$f_1,\ f_2,\ f_3 > 0$$

$$C_1 \geq C_2 \geq C_3$$

The reason for this rather complicated scheme is that the use of only one general exponential distribution

$$f(x) = \begin{cases} -\lambda\exp\left(-\lambda(x-a)\right), & x>a, \quad \lambda>0 \\ 0 & , \quad x<a \end{cases} \tag{27}$$

generates values that, in general, may be too near the value of a, and the previous distribution generates first costs for the proposals which can be considered as corresponding to small, medium and large size proposals. The particular values used were:

$$C_0 = 15{,}000 \tag{28}$$

$a_1 = 6{,}000$

$C_1 = 11{,}000$

$a_2 = 10{,}000$

$C_2 = 15{,}000$

$a_3 = 14{,}000$

$C_3 = 19{,}000$

This distribution was used throughout the study.

Measures of Effectiveness

Total Capital at Time t

The objective of the MPV criterion is stated to be the maximization of a firm's capital at a horizon date. The total capital of a firm at time t is defined as the present worth at time t of the future receipts of unliquidated investments that were made on or before time t (2, pp. 110-112). The following scheme of measure of total capital is developed in the cited reference. A proposal "k" with growth rate g_k undertaken at time T has an unliquidated capital

$$S_k = \sum_{t=T+1}^{\infty} S_{kt}(1+g_k)^{T-t} \tag{29}$$

For the same proposal at a time $t_1 > T$, the theoretical amount of capital that remains invested in this proposal is:

Unliquidated capital of proposal at period t_1 =

$$\sum_{t=t_1+1}^{t=\infty} S_{kt}(1+g_k)^{t_1-t} \tag{30}$$

Under this condition in the computer simulation, each proposal "k" that is undertaken by the firm at a period t is classified by its growth rate g_k, and its cash flows are lumped with the unliquidated cash flows of proposals with the same growth rate undertaken in period t or before. The lumping is made in a matrix of unliquidated cash flows $G(g_k,t+1)$, $G(g_k,t+2),\ldots,G(g_k,\infty)$.

The unliquidated capital invested in proposals with growth rate g_k at time T will be:

$$\text{Unliquidated capital at } g_k = \sum_{t=T+1}^{t=\infty} G(g_k,t)(1+g_k)^{T-t} \tag{31}$$

and the total capital will be:

$$C_T = \text{Total capital at time T} = \sum_{K} \sum_{t=T+1}^{t=\infty} G(g_k,t)(1+g_k)^{T-t} \tag{32}$$

where the K in the first summation stands for all the classes defined by g_k.

For this work the classes were defined for values of $g_k = i_\delta$, $i_\delta + .01,\ldots,.30$, where .30 was the highest prospective growth rate obtainable.

Average Growth Rate of Capital

If the total capital of the firm at a horizon date C_H is considered to be $(1+\bar{g})^H$ times the initial capital M_0, a monotonic measure of the growth of capital is $\bar{g}$, the *average growth rate of capital*

$$\bar{g} = \sqrt[H]{\frac{C_H}{M_0}} - 1 \qquad (33)$$

The average growth rate of capital was the measure of effectiveness used in this work, since the results presented have easier representations in terms of average growth rates than in terms of capital at a horizon date.

Process Followed in the Computer Simulation

The work was carried out with three Fortran IV programs. The essential features of them will be presented, omitting all the parts which deal with the collection of complementary data.

From the conditions stated in the previous sections of this chapter, it is obvious that for our purposes an investment proposal k can be completely described if the following factors are defined:

1. period of generation "t",
2. initial cost S_{0k},
3. cash flow pattern (Single Payment, Uniform Series, Increasing Series or Decreasing Series),
4. growth rate g_k,
5. life of the proposal Lk.

A set of investment proposals for any period can be easily generated through random sampling in the distributions that define an Investment Situation.

Definition of Company i, $C_H(i,m)$ and $g(i,m)$

From here on, a *company i* will be defined as the set of:

1. an initial capital M_0,
2. an interest rate in highly liquid funds i_δ,
3. a horizon date H,
4. a set of H + 1 sets of Investment Proposals for successive periods of time, t = 0,1,2,...,H.

The capital at a horizon date for the company, resulting from the successive application period by period of the MPV criterion to the SIP(t), for t = 0,1,2,...,H, with a tentative marginal growth rate m, will be termed $C_H(i,m)$.

The term g(i,m) is defined to be:

$$g(i,m) = \sqrt[H]{C_H(i,m)/M_0} - 1 \tag{34}$$

The application of the MPV criterion is made under the assumptions of internally generated funds for capital investments and the MPV was used as the only criterion for selection.

Process of Determination of $\bar{m}^*$ for an Investment Situation

In this process four phases can be isolated.

1. Define the Investment Situation.

2. Generate companies (replications) through computer random sampling. These companies are indexed through the parameter i, and $i = 1,2,\ldots,R$.

3. Obtain values of capital at horizon year $C_H(i,m)$ for each company, and for different values of m. Compute the values of $g(i,m)$ and accumulate the pairs of values $(m,g(i,m))$ for $i = 1,2,\ldots,R$, $m = i_\delta, i_\delta + .02,\ldots,.26$.

4. Determine $\bar{m}^*$ from the accumulated pairs $(m,g(k,m))$.

Definition of Investment Situations

An investment situation is characterized by specifying probability distributions for the parameters that define the basic types of investments to be considered. These parameters include the initial capital, the interest rate on highly liquid funds, the horizon date, the number of investment proposals to be considered each period, and the initial cost, life, growth rate, and cash flow pattern of the individual proposals.

The values of the parameters of the investment situations under study are summarized in Tables 3 and 4 appearing later in Chapter V.

Generation of a Company

A company can be generated through Monte Carlo sampling in the distributions of an investment situation through the following sequence:

1. For each company:

   a. Read the parameters that define the investment situation under study.

2. For each period t, $t = 0,1,\ldots,H$:

a. Generate through Monte Carlo sampling of the distribution of number of proposals per period, the number of proposals to be considered in period t, n(t).

3. For each proposal j in the period "t", j = 1,2,...,n(t):

   a. Obtain through Monte Carlo sampling on the statis distribution of cost of proposals, the uncorrected cost of proposal "j" S^*_{0j}.

   b. Obtain the corrected cost of proposal "j".

$$S_{0j} = S^*_{0j}(1+gp)^t \tag{35}$$

   c. Obtain through Monte Carlo sampling in the cash flow pattern distribution the cash flow pattern of the proposal j.

$$CF_j = SP,\ US,\ IS,\ \text{or}\ DS$$

   d. Obtain through Monte Carlo sampling in the growth rate distribution a growth rate gj.

   e. Record in a magnetic tape the resulting values plus identifiers for the company (k), period (t), number of proposals per period (n(t)) and proposal (j).

The last set of values is attached to the values of S_{0j}, CF_j, gj, for identification purposes.

It must be noted that the indispensable factors for the generation of the H + 1 sets of investment proposals (SIP(T), t = 0,...,H) are:

1. the value of H,
2. distribution related to a number of proposals to be considered per period,
3. distribution related to cash flow shape composition,
4. distribution related to growth rates,
5. distribution related to life of the proposals,
6. static distribution of first cost,
7. growth rate of first cost gp.

It was considered better to study the effect of different values of i_δ over the same sets of proposals. Because of this, the investment situations with identical characteristics in the seven factors just mentioned (but differing in levels of capital and values of i_δ) share the R sets of H + 1 sets of investment proposals for t = 0,1,2,...,H.

Computation of Values of $C_H(i,m)$ and $g(i,m)$

Each company k of the R companies studied in an investment situation, together with a tentative marginal growth rate m, determines a value of capital at a horizon date for company k, using $m(C_H(i,m))$ through the following process:

1. For each period t, t = 0,1,2,...,H-1.
   a. Obtain the value of the budget at hand B(T); this budget is defined to be M_0 for t = 0.
   b. Read from a magnetic tape the number of proposals to be considered in period t, n(t).

c. For each proposal k, $k = 1,2,\ldots,n(t)$, read from tape the values of life, L_k, prospective growth rate g_k, initial cost S_{0k}, and the cash flow pattern of proposal CF_k. From these data obtain the cash flows $S_{0k}, S_{1k}, \ldots, S_{Lk}$ for proposal k, and calculate the Prospective Value of the Proposal k, $P_k(m)$ where,

$$P_k(m) = \sum_{t=1}^{t=\infty} S_{tk}(1+m)^{-t} + \frac{1 + i_\delta}{1 + m} S_{0k} \tag{36}$$

d. Construct the integer programming formulation for the MPV criterion

$$\text{Max } Z = \sum_{k=1}^{k=n(t)} X_k P_k(m) \tag{37}$$

subject to

$$\sum_{k=1}^{k=n(t)} S_{0k} X_k + B(t) \geq 0$$

and X_k = 0 if proposal k is rejected

1 if proposal k is accepted.

e. Obtain an optimum solution $X = (X_1, X_2, \ldots, X_{n(t)})$.

f. For each accepted proposal k, add the prospective cash flows $S_{1k}, S_{2k}, \ldots, S_{Lk}$ to:

1. The prospective cash flows receivable from investments at growth rate g_k, in the periods $t+1, t+2, \ldots, t+L_k$. These prospective cash flows are recorded in a matrix G(gk,t) to be used for the computation of values of total capital.

2. To a vector $B(s)$, for $s = t+1, t+2, \ldots, H$ from which the value of the budgets is determined.

g. Add $(1+i_\delta)$ times the value of $\left(B(t) - \sum_{k=1}^{k=n(t)} S_{0k}X_k\right)$ to $B(t+1)$.

h. Calculate the value of total Capital at period t, $C_t(i,m)$ as prescribed in page 50.

2. For period H.

a. Perform steps a through h under 1.

b. Obtain the average growth rate of company k using tentative growth rate m:

$$g(i,m) = \sqrt[H]{\frac{C_H(i,m)}{M_0}} - 1 \tag{38}$$

<u>Search for $\bar{m}^*$</u>

Each value of g(i,m) can be considered as a random sample from a universe of g(m). g(m) is a continuous variable with some associated probability function. The variability observed in the obtained samples of g(i,m) required the use of replications. In this case the number of replications (companies) was eight throughout the study for all investment situations; this number was decided after some experimentation and seems to provide a margin of error small enough for qualitative analysis.

The probability function related to g(m) for an investment situation changes with m and has some expected values $E\{g(m)\}$. We are interested in the search of the point $\bar{m}^*$ that maximizes $E\{g(m)\}$ over the interval $i_\delta < m < \infty$.

The experimental results seem to confirm that $E\{g(m)\}$ present a unimodal maximum at $\bar{m}^*$, $i_\delta < \bar{m}^* < g$.

For each investment situation and each m ($m = i_\delta, i_\delta + .02, i_\delta + .04, \ldots, .26$) the previous steps rendered eight values for $\bar{g}(i,m)$, $i = 1,2,\ldots,8$. The averages of these values were obtained, $\bar{g}(m)$, and a linear regression to a fourth degree polynomial form with respect to m was made in order to obtain a smoothed curve. The form used was:

$$f(m) = A_0 + A_1 m^1 + A_2 m^2 + A_3 m^3 + A_4 m^4 \quad (39)$$

Over this smoothed curve, the highest point was found and the m^* obtained was rounded to the nearest .01 and recorded as the $\bar{m}^*$ for the respective investment situation.

CHAPTER V

ANALYSIS OF THE RESULTS, CONCLUSIONS AND RECOMMENDATIONS

Investment Situations

This sections consists mainly of Tables 3 and 4 which present the characteristics of the investment situations investigated.

Table 3 sets forth the values of the variables that were kept constant as parameters throughout the simulation study, and Table 4 explains the variable characteristics of each investment situation.

Table 3. Parameters for the Investment Situations Studied

FACTOR:	VALUE OF PARAMETERS
Horizon Date H	H = 15
Probability distribution of first cost of proposals (static)	C_0 = 15,000 a_1 = 6,000 C_1 = 11,000 a_2 = 10,000 C_2 = 15,000 a_3 = 14,000 C_3 = 19,000
Probability distribution related to to life of proposals	a = 2 λ = 3
Probability distribution of growth rate of proposals	See Table 2 and Figure 1
Initial Capital	Set at four levels: $1\text{x}10^5$, $2\text{x}10^5$, $3\text{x}10^5$ and $4\text{x}10^5$

Table 4. Factors that Define the Investment Situations

Class of Investment Situations	i_δ	Cash Flow Shape Distribution				Distribution for Proposals per Period	
		P_{US}	P_{SP}	P_{IS}	P_{DS}	n	σ_n
A	.04	1.0	0.0	0.0	0.0	10	0
B	.04	0.0	1.0	0.0	0.0	10	0
C	.04	0.0	0.0	1.0	0.0	10	0
D	.04	0.0	0.0	0.0	1.0	10	0
E	.04	1.0	0.0	0.0	0.0	10	2
F	.04	0.0	1.0	0.0	0.0	10	2
G	.04	0.0	0.0	1.0	0.0	10	2
H	.04	0.0	0.0	0.0	0.0	10	2
I	.10	1.0	0.0	0.0	0.0	10	2
J	.10	0.0	1.0	0.0	0.0	10	2
K	.10	0.0	0.0	1.0	0.0	10	2
L	.10	0.0	0.0	0.0	1.0	10	2
M	.04	1.0	0.0	0.0	0.0	10	4
N	.04	0.0	1.0	0.0	0.0	10	4
O	.04	0.0	0.0	1.0	0.0	10	4
P	.04	0.0	0.0	0.0	1.0	10	4
Q	.04	1/5	4/5	0.0	0.0	10	0
R	.04	2/5	3/5	0.0	0.0	10	0
S	.04	3/5	2/5	0.0	0.0	10	0
T	.04	4/5	1/5	0.0	0.0	10	0
U	.04	1/2	1/2	0.0	0.0	10	0
V	.04	0.0	1/2	1/2	0.0	10	0
W	.04	0.0	1/2	0.0	1/2	10	0
X	.04	1/2	0.0	1/2	0.0	10	0
Y	.04	1/2	0.0	0.0	1/2	10	0
Z	.04	0.0	0.0	1/2	1/2	10	0
AA	.04	1/6	1/2	1/6	1/6	10	0
AB	.04	1/6	1/6	1/2	1/6	10	0
AC	.04	1/2	1/6	1/6	1/6	10	0
AD	.04	1/6	1/6	1/6	1/2	10	0
AE	.04	1/4	1/4	1/4	1/4	10	0
AF	.10	1/6	1/6	1/2	1/6	10	0
AG	.10	1/6	1/2	1/6	1/6	10	0
AH	.10	1/6	1/6	1/6	1/2	10	0
AI	.10	1/2	1/6	1/6	1/6	10	0

Investment situations which differ only in the values of initial capital are designated with the same letters, followed by a number r ($r = 1,2,3,4$) to express the level of capital used since the levels of M_0 used were 10^5, 2×10^5, 3×10^5 and 4×10^5 for all the combinations used for the other factors.

Each group of four investment situations bearing the same identification letters will be called a *class* of investment situations.

Before proceeding with the analysis of results, a digression will be made to introduce some terms and concepts necessary for the following discussion.

Relationships Between the Marginal Growth Rate and the Rate of Budget to Total First Cost of Proposals

Suppose that a firm has the curve of investment opportunities represented by Figure 7. Also suppose that the average ratio of budget to total first cost of proposals* for the same period has a value of .40. Under these circumstances it can be expected that the marginal growth rate for the enterprise has a value of .13. These values can easily be obtained from Figure 7, page 44.

The value of .4 for the ratio of budget to total first cost of proposals may imply that the firm can finance on the average 40 per cent of the proposals received, and the growth rate of the marginal investments would be in the neighborhood of .13.

---

*In the ensuing discussions the average ratio of budget to total first cost of proposals will be referred to as ABCP, and the ratio of budget to total cost of proposals will be named BCP.

For the investment situations presented in this study, the decision with respect to an investment proposal is either "accept" or "reject". The adoption of fractions of proposals is not allowed. This restriction causes large variations in the values of the marginal growth rates from period to period. In the following discussion, a range of values for the marginal growth rates will be mentioned to express the fact that the marginal growth rate changes with time.

Analysis of Results

Appendix A presents an example of the general type of results obtained for each investment situation: an investment situation was chosen, and Appendix A presents in graphical form its values of $\bar{g}(m)$, the regression curves used and the values of $\bar{m}^*$ obtained.

Appendix B presents tables showing the values of $\bar{m}^*$ and $\bar{g}(\bar{m}^*)$ for particular investment situations. In the following discussions, only the values of $\bar{m}^*$ are mentioned.

Effect of Interest Rate Earned on Highly Liquid Funds

Some tests were made for several investment situations with the intention of revealing any shift in the values of $\bar{m}^*$ for values of $i_\delta = .04$ and $i_\delta = .05$. For these small variations in i_δ, no noticeable effect on $\bar{m}^*$ was observed.

In an extreme test the value of i_δ was set at .10 for the investment situations included in the classes I, J, K, L, AF, AG, AH and AI. These classes of investment situations used the same sets of investment proposals as the investment situations included in the classes E, F, G, H, AB, AA, AD and AC, respectively, but the latter classes had an i_δ set at .04.

The resulting values for $\bar{m}^*$ appear in Table 5.

Table 5. $\bar{m}^*$ of Investment Situations Differing only in Values of i_δ

Class of Investment Situations	i_δ	Initial Capital			
		10^5	2×10^5	3×10^5	4×10^5
E	.04	.17	.12	.10	.08
I	.10	.16	.15	.14	.14
F	.04	.19	.17	.15	.13
J	.10	.20	.18	.15	.14
G	.04	.17	.15	.12	.10
K	.10	.18	.14	.14	.14
H	.04	.14	.10	.08	.07
L	.10	.13	.12	.13	.13
AA	.04	.19	.16	.12	.11
AG	.10	.17	.15	.15	.14
AB	.04	.18	.14	.12	.10
AF	.10	.18	.15	.13	.14
AC	.04	.17	.12	.09	.07
AI	.10	.15	.12	.14	.15
AD	.04	.17	.13	.10	.09
AH	.10	.18	.14	.12	.13

The investment situations are ordered by pairs to make the comparison easier. Investment situations E1 and I1 differ only in the value of i_δ employed (.04 and .10, respectively), and the same is valid for the pairs E2 and I2, etc.

It can be observed for the investment situations with an initial capital of \$100,000 that there is no consistent tendency for the values of $\bar{m}^*$ to change with values of i_δ. It must be explained that for these investment situations the average ratios of budget to total cost of proposals (ABCP) were small, and, in general, the fraction of budget invested in proposals with growth rates from .04 to .10 was very small. Because of this, a change in the value of i_δ from .04 to .10 did not affect profoundly the values of $\bar{m}^*$ [see Figures 8(a) and 8(b)].

For investment situations with i_δ = .10 and initial capitals of \$200,000, \$300,000 and \$400,000, it can be observed that the values of $\bar{m}^*$ are greater than or equal to the $\bar{m}^*$'s for the corresponding investment situations using i_δ = .04.

The reason is that for these levels of initial capital, the average ratios of budget to total first cost of proposals (ABCP) were comparatively large. Under these circumstances, most of the marginal investments for the firms using i_δ = .04 had growth rates in the range $.04 \leq gj \leq .10$. A value of i_δ = .10 excludes from further consideration proposals in this range and renders higher values of $\bar{m}^*$. [see Figures 9(a) and 9(b)].

The magnitude of the shifts in values of $\bar{m}^*$ with a change in the value of i_δ from .04 to .10 depends on the value of $\bar{m}^*$ for the situation with i_δ = .04. For example, investment situation F2, F3 and F4 used i_δ = .04 and had values of $\bar{m}^*$ of .17, .15 and .13, respectively. These values are considerably higher than .10. The same sets of proposals using a value of .10 constituted investment situations J2, J3 and J4,

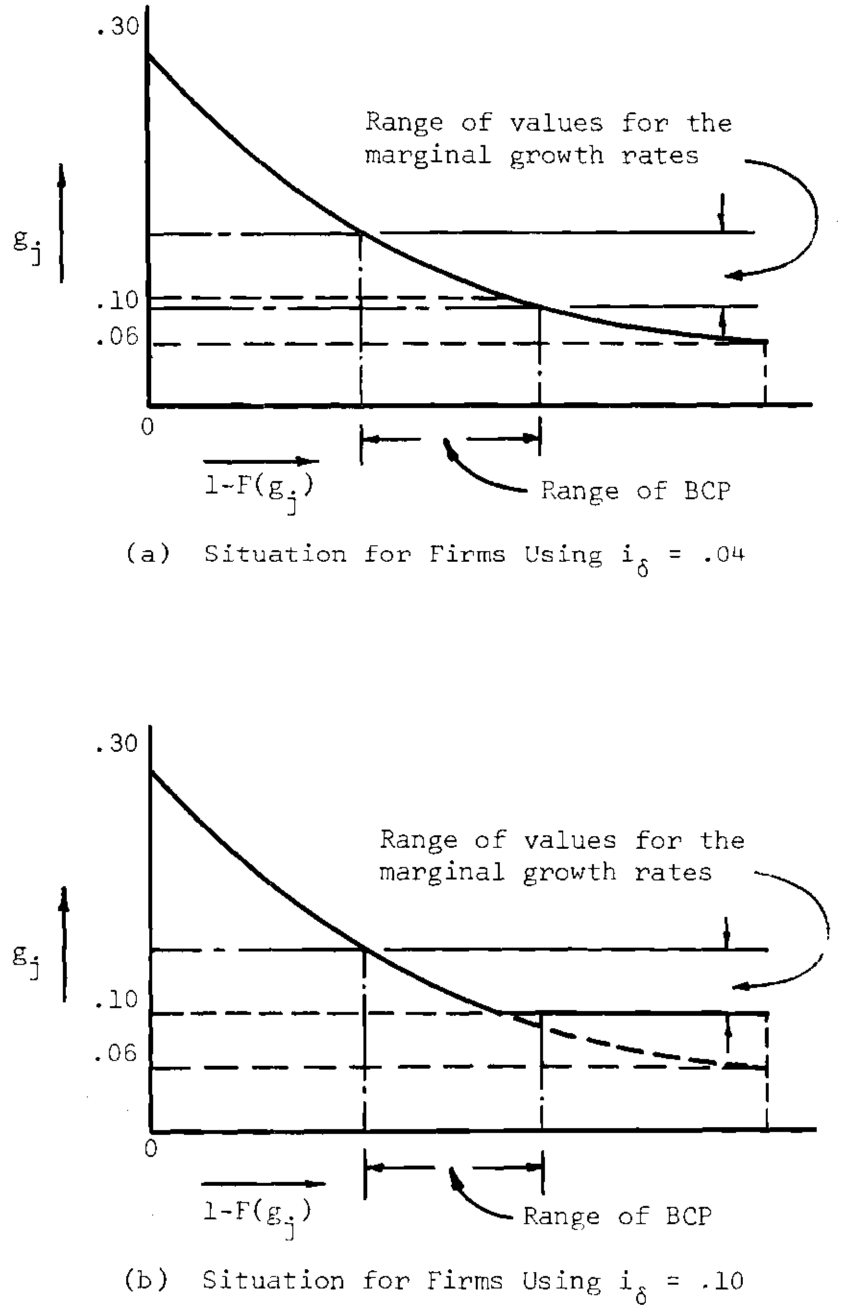
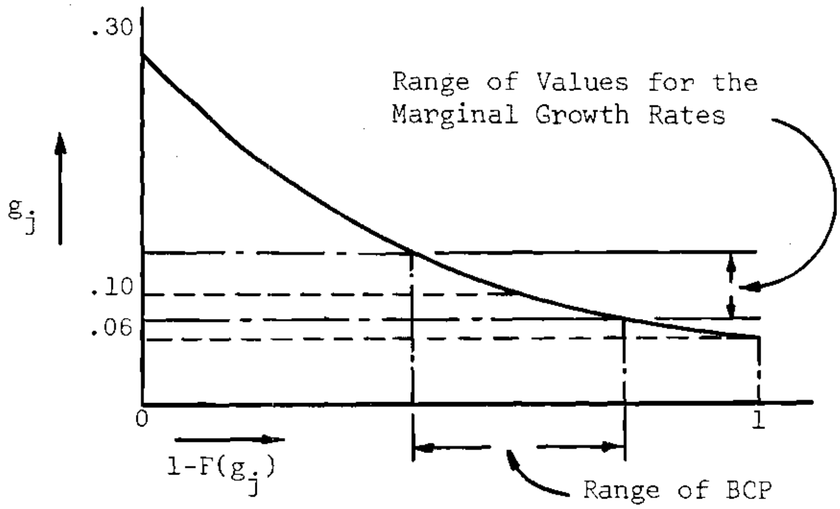
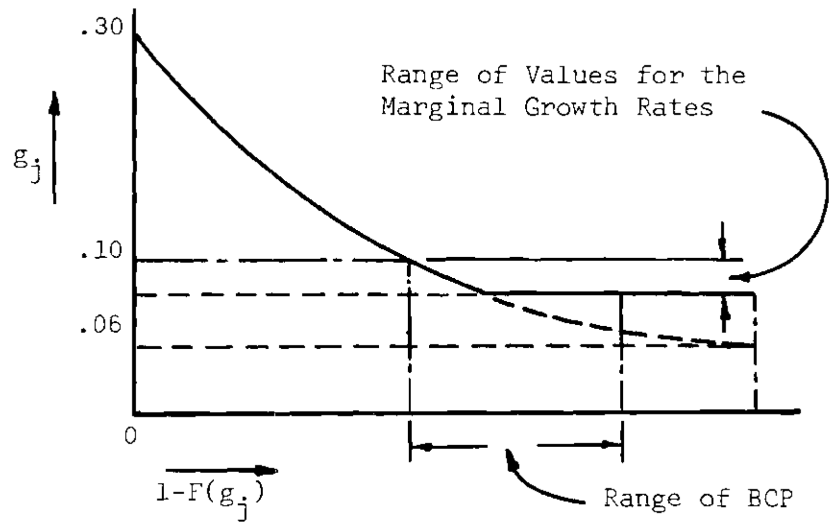


Figure 8. Idealized Ranges for the Values of the Marginal Growth Rates for the Investment Situations Using an Initial Capital Condition of \$100,000



(a) Situation for Firms with $i_\delta = .04$



(b) Situation for Firms with $i_\delta = .10$

Figure 9. Idealized Ranges of the Values of the Marginal Growth Rates for the Investment Situations Using Initial Capitals Greater than $200,000

with values of $\bar{m}^*$ of .18, .15 and .14. The values of the shifts in $\bar{m}^*$ were +.01, .00 and +.01, respectively.

Investment situations AB2, AB3 and AB4 present values of $\bar{m}^*$ of .14, .12 and .10 using an $i_\delta = .04$. These values for $\bar{m}^*$ are close to .10. Using an $i_\delta = .10$, the same investment proposals constituted investment situations AF2, AF3 and AF4, with values of $\bar{m}^*$ of .15, .13 and .14, showing shifts in the values of $\bar{m}^*$ of +.01, +.01 and +.04.

It is apparent that the value of the shift is more important in the cases in which $\bar{m}^*$ is close to .10 in investment situations with $i_\delta = .04$.

The value of i_δ sets a lower limit to the range of values of the marginal growth rates, which in turn defines the values of $\bar{m}^*$. In the investment situations studied with $i_\delta = .10$, the lower bounds seemed to range around .12; i.e., no investment situation with $i_\delta = .10$ presents a $\bar{m}^*$ lower than .12.

Effect of Variability in the Number of Proposals to be Considered Each Period

This parameter was investigated at three levels of variability.

The type of statistical distribution related to the number of proposals per period was normal. The mean was set at a value of 10 and the standard deviations were set at 0, 2 and 4 for different investment situations.

The results are summarized in Table 6 in which the investments are grouped in blocks of three; the investment situations in each block differ only in the variance used for the distribution of the number of proposals to be considered each period.

Table 6. $\bar{m}^*$ for Groups of Three Investment Situations that Differ Only in Variability in the Number of Proposals per Period

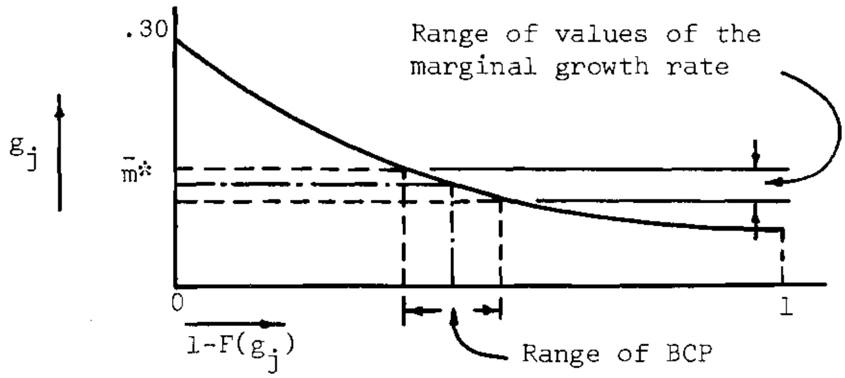
Class of Investment Situations	Initial Capital			
	10^5	2×10^5	3×10^5	4×10^5
A	.17	.12	.09	.08
E	.17	.12	.10	.08
M	.16	.13	.10	.08
B	.20	.17	.17	.14
F	.19	.17	.15	.13
N	.20	.15	.13	.12
C	.18	.15	.12	.11
G	.17	.15	.12	.10
O	.18	.15	.13	.11
D	.15	.10	.09	.06
H	.14	.10	.08	.07
P	.13	.09	.08	.07

There is no noticeable tendency in changes in the values of $\bar{m}^*$ with changes in variance.

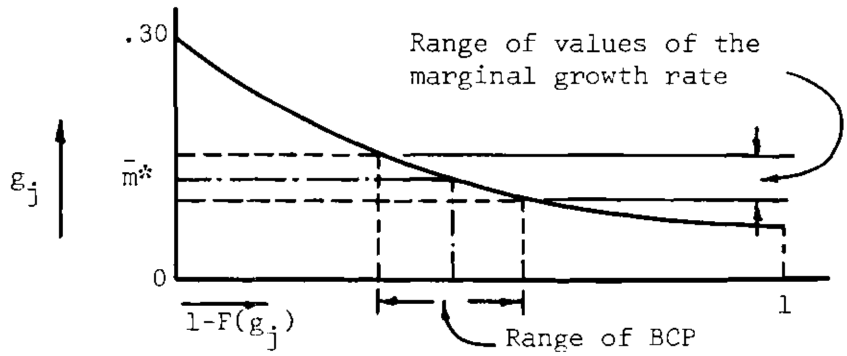
Some increase in the values of $\bar{m}^*$ was expected and its absence is attributed to an averaging effect on the marginal growth rates for the different periods of operation of a firm; i.e., the range of values for the marginal growth rates increased but the average marginal growth rate did not suffer a noticeable change [see Figures 10(a) - (c)].

Effect of Cash Flow Pattern Composition

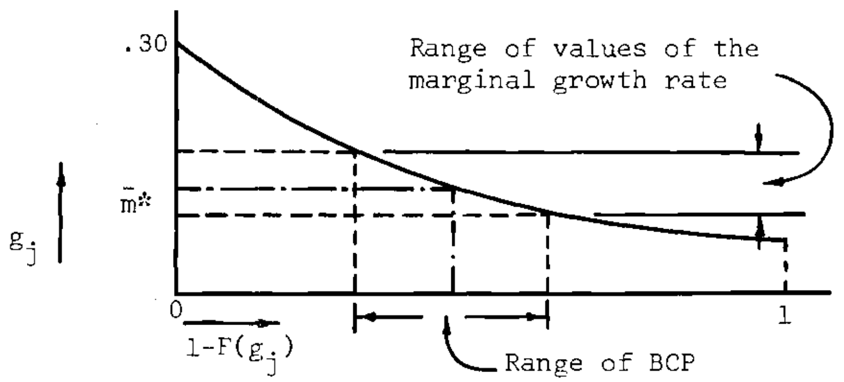
There is some effect from the cash flow pattern composition over the values of $\bar{m}^*$. The study of the effects of this factor was carried over in three parts. The pertinent values of $\bar{m}^*$ can be found in Table 7.



(a) Situation for σ = 0



(b) Situations for σ = 2



(c) Situation for σ = 4

Figure 10. An Idealized Representation of the Effect of Variability on the Range of Marginal Growth Rates

Table 7. Values of $\bar{m}^*$ for Different Cash Flow Pattern Compositions

Class of Investment Situations	Initial Capital			
	10^5	2×10^5	3×10^5	4×10^5
A	.17	.12	.09	.08
B	.20	.17	.17	.14
C	.18	.15	.12	.11
D	.15	.10	.09	.06
Q	.18	.17	.14	.13
R	.19	.17	.13	.11
S	.18	.14	.12	.10
T	.17	.13	.11	.09
J	.19	.15	.12	.10
V	.19	.15	.13	.10
W	.18	.14	.13	.11
X	.19	.14	.12	.09
Y	.17	.12	.09	.07
Z	.18	.11	.10	.08
AA	.19	.16	.12	.11
AB	.18	.14	.12	.10
AC	.17	.12	.09	.07
AD	.17	.13	.10	.09
AE	.18	.15	.11	.10

1. Investment situations in the classes A, B, C and D were constructed. The factors that define these four situations differed only in the cash flow pattern composition since class A used only Single Payment cash flow pattern proposals, and classes B, C and D used proposals with cash flow patterns of the types Uniform Series, Increasing Series and Decreasing Series. If the $\bar{m}^*$ related to certain cash flow pattern is denoted $\bar{m}^*$ (cash flow pattern), then it can be seen that:

$$\bar{m}^*(SP) \geq \bar{m}^*(IS) \geq \bar{m}^*(US) \geq \bar{m}^*(DS)$$

in equality of initial capital.

2. Investment situations in the classes Q, R, S and T were constructed with probabilities of a Single Payment cash flow pattern of 0.8, 0.6, 0.4 and 0.2, respectively, and probabilities of Uniform Series cash flow pattern of 0.2, 0.4, 0.6 and 0.8. The resulting values of $\bar{m}^*$ are displayed in Table 8. The existence of some relationship is easily observed since the values for $\bar{m}^*$ are higher for investment situations with higher proportions of Single Payment cash flow pattern proposals.

Table 8. Values of $\bar{m}^*$ for Investment Situations with Different Proportions of Single Payment and Uniform Series Cash Flow Patterns

Class of Investment Situation	Initial Capital			
	10^5	1×10^5	3×10^5	4×10^5
B	.20	.17	.17	.14
Q	.18	.17	.14	.13
R	.19	.17	.13	.11
S	.18	.14	.12	.10
T	.17	.13	.11	.09
A	.17	.12	.09	.08

3. In order to have some complementary data, investment situations in the classes U through AE were constructed with different proportions of the four basic cash flow patterns. With few exceptions the results confirmed the existence of some functional relation of $\bar{m}^*$ with the cash flow shape composition of the investment situations. The results can be seen in Table 7.

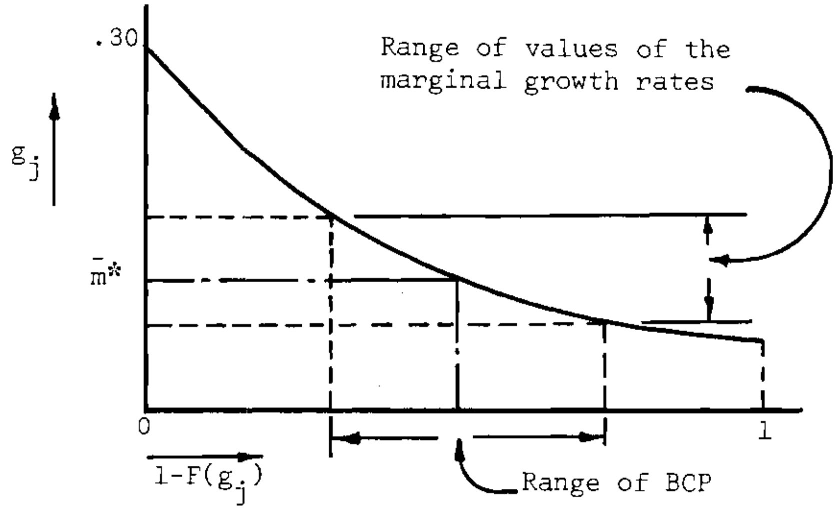
From the general results it can be observed that investment situations with cash flow pattern composition having large amounts of Single Payment or Increasing Series proposals present values of $\bar{m}^*$ higher than situations with large proportions of Decreasing Series or Uniform Series proposals.

Since the investment situations with Single Payment proposals show the highest values of $\bar{m}^*$ and situations with Decreasing Series proposals present the lowest values of $\bar{m}^*$, the following situation will be centered on these two extreme cases. It must be remembered that $\bar{m}^*$ is defined to be the average growth rate at which the differential cash flows of the best and next best decisions are invested (1, p. 158).

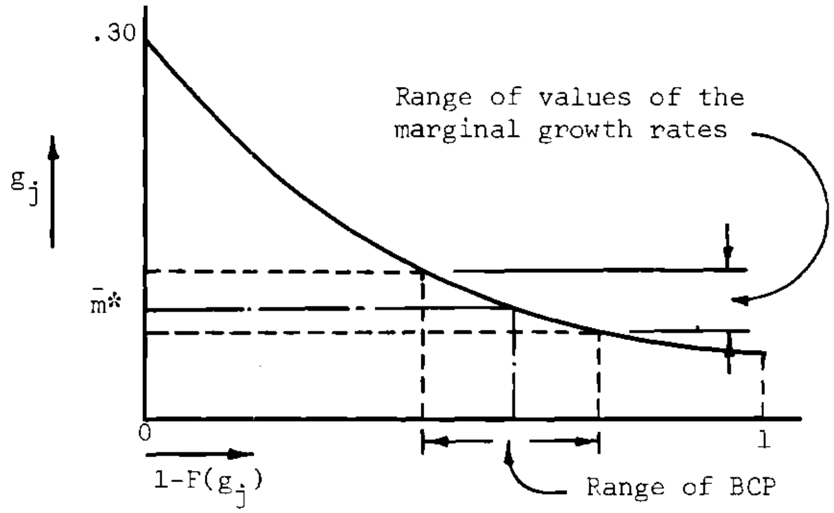
In an investment situation with Single Payment cash flow pattern proposals, the selection of a proposal j at period t in place of a proposal k can alter significantly the amounts and timing of the cash flows for the firm in subsequent periods, possibly leading to a complete change in the future decisions of the firm.

The differential cash flows would then be invested at an average rate near the growth rate of capital ($\tilde{g}$). The range of variation of the marginal growth rates for this investment situation is very broad, and this, in turn, will result in higher values for $\bar{m}^*$. An idealized picture is presented in Figure 11(a).

An investment situation with decreasing series cash flow pattern proposals presents a different situation. The decision to undertake proposal j in place of proposal k at a period t will, in general, affect to a smaller degree the future decisions that the firm will undertake,



(a) Situation with a Large Probability of Single Payment Cash Flow Pattern Proposals



(b) Situation with a Large Probability of Decreasing Series Cash Flow Pattern Proposals

Figure 11. An Idealized Representation of the Effect of Different Cash Flow Patterns on the Range of Values of the Marginal Growth Rates

since the decreasing series cash flow pattern proposals yield revenues in a stream decreasing in time.

The size of the change on the budgets would be small, and, in general, the differential cash flows of the next and next best decisions would be invested at lower growth rates in a small range of marginal growth rates. The values of $\bar{m}^*$ are consequently lower. An idealized picture is shown in Figure 11(b).

Conclusions

1. The interest rate on highly liquid funds, i_δ, places a lower boundary to the range of values of the marginal growth rates of a firm. The values of the average marginal growth rate $\bar{m}^*$ become larger with increments in the values of i_δ, but this effect becomes noticeable only in the neighborhood of i_δ. For the investment situations studied at $i_\delta = .10$, the lowest value of $\bar{m}^*$ was .12. Investment situations using $i_\delta = .04$ and with values of $\bar{m}^*$ of .12 or lower, show shifts of $\bar{m}^*$ to values ranging from .12 to .14. Investment situations with $\bar{m}^*$ of .13 and .14 with $i_\delta = .04$ increased their values of $\bar{m}^*$ by .01 or .02 with an increment in i_δ from .04 to .10. Finally, investment situations with $\bar{m}^*$ higher than .14 with $i_\delta = .04$ did not present a noticeable pattern of change.

2. The variability in the number of proposals to be considered each period had no noticeable effect on the values of $\bar{m}^*$.

3. The cash flow pattern composition presents a definite effect on the values of $\bar{m}^*$. This effect is explained in terms of the magnitude in change of future decisions caused by the acceptance or rejection of

investment proposals with different cash flow patterns.

For investment situations using proposals with cash flow pattern:

a. Single Payment (SP).

b. Increasing Series (IS).

c. Uniform Series (US).

d. Decreasing Series (DS).

the following relationships in the values of $\bar{m}^*$ were observed:

$$\bar{m}^*(SP) \geq \bar{m}^*(IS) \geq \bar{m}^*(US) \geq \bar{m}^*(DS)$$

For the investment situations using only Single Payment cash flow pattern proposals, the values of $\bar{m}^*$ are from .05 to .08 larger than the values of $\bar{m}^*$ observed for investment situations with the same level of initial capitals, but using only Decreasing Series cash flow pattern proposals.

Recommendations for Further Research

1. The measure of an investment's worth by the MPV criterion (the prospective value of a proposal's worth, p. 24) favors short life proposals, assigning them higher values. In this study, a general exponential distribution was used for the life distributed of the investment proposals (see p. 45).

It would be interesting to investigate the effectiveness of the MPV criterion using some other forms of distribution that may assign little probability to proposals with short lives. Under the same situation, the behavior of $\bar{m}^*$ would be investigated.

2. A relationship was found among the *average rates of budget to total first cost of proposals per period ABCP* and the values of the *tentative average marginal growth rates m*. In the range of m studied ($.04 \leq m \leq .26$), the values of ABCP were found to increase monotonically with m. An example of this effect is presented in Figure 12 for the investment situations AE1, AE2, AE3 and AE4.

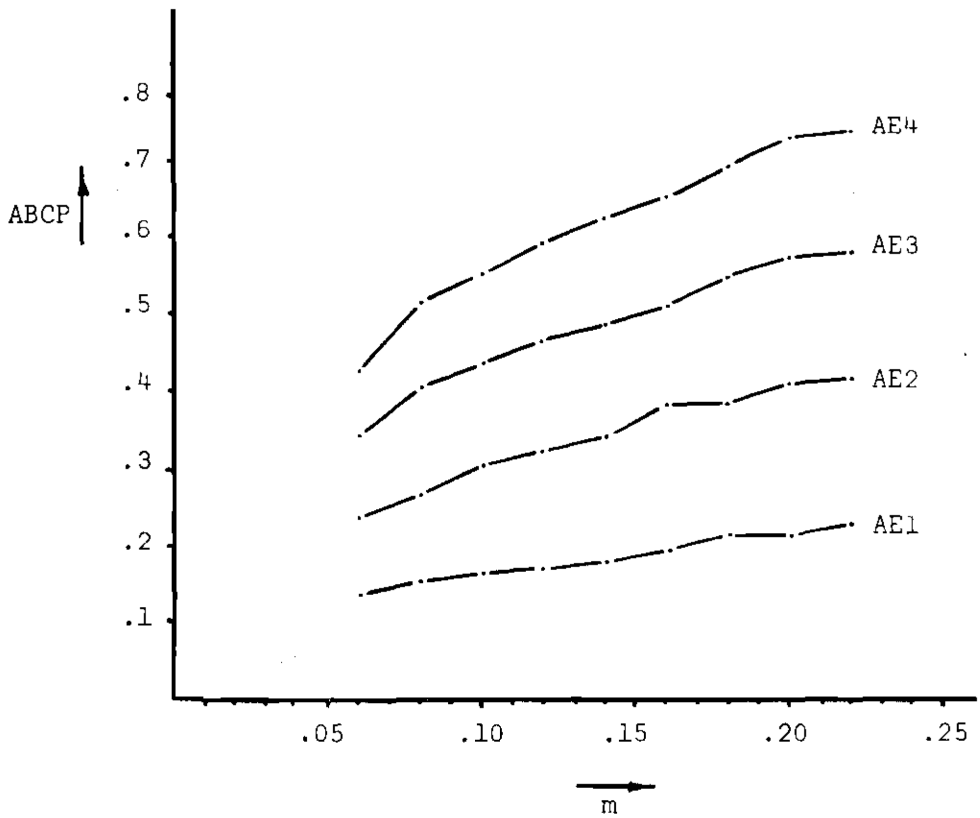


Figure 12. Relationship of ABCP and m for Investment Situations AE1, AE2, AE3 and AE4

When the values of ABCP are plotted on a graph of the curve of investment opportunities using m as ordinate and ABCP as abscissa, the *ordinate* of the intersection of the curve of ABCP and the curve of

Figure 13 presents the plots of the values of ABCP for investment situations AE1, AE2, AE3 and AE4. Table 9 sets forth the corresponding values, and Tables 10 and 11 present values of $\bar{m}^*$ and the ordinate of the intersection for some investment situations, respectively.

It would be worthwhile to investigate the generality of this phenomenon.

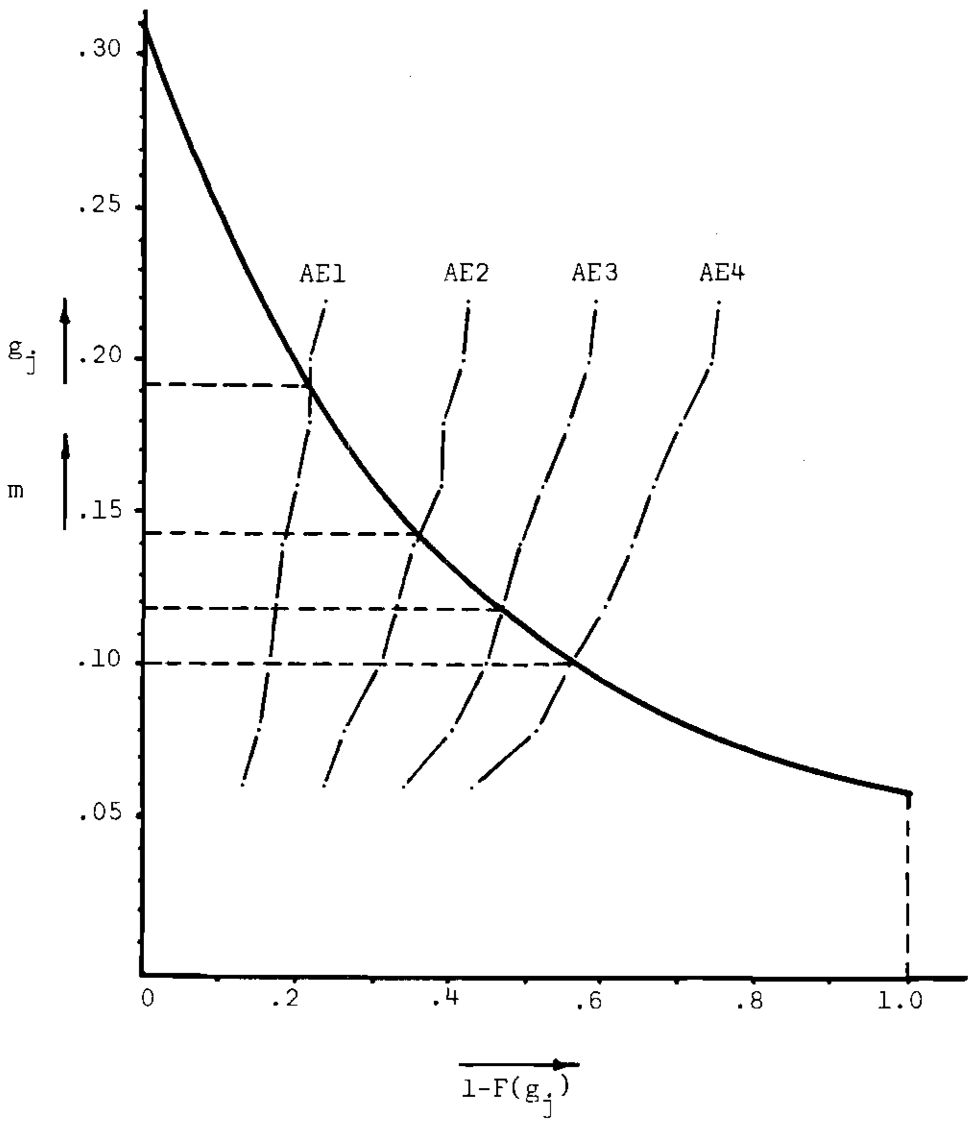


Figure 13. Intersections of Curves of ABCP with the Curve of Investment Opportunities

Table 9. Values of ABCP for Investment Situations Class AE for Different Values of m

m	Initial Capital 10^5	2×10^5	3×10^5	4×10^5
.06	.129	.235	.342	.430
.08	.148	.264	.405	.514
.10	.160	.303	.438	.552
.12	.167	.320	.464	.593
.14	.177	.343	.485	.629
.16	.194	.371	.512	.657
.18	.210	.384	.545	.696
.20	.212	.408	.574	.733
.22	.217	.417	.580	.744

Table 10. Values of $\bar{m}^*$ for Some Investment Situations

Class of Investment Situations	Initial Capital 10^5	2×10^5	3×10^5	4×10^5
A	.17	.12	.09	.08
B	.20	.17	.17	.14
C	.18	.15	.12	.11
D	.15	.10	.09	.06
U	.19	.15	.12	.10
V	.19	.15	.13	.10
W	.18	.14	.13	.11
X	.19	.14	.12	.09
Y	.17	.12	.09	.07
Z	.18	.11	.10	.08
AA	.19	.16	.12	.11
AB	.18	.14	.12	.10
AC	.17	.12	.09	.07
AD	.17	.13	.10	.09
AE	.18	.15	.11	.10

Table 11. Values of the Abscissas at the Intersections of ABCP and the Curve of Investment Opportunities Opportunities for Some Investment Situations

Class of Investment Situations	Initial Capital			
	10^5	2×10^5	3×10^5	4×10^5
A	.165	.12	.100	.084
B	.212	.169	.140	.124
C	.180	.140	.116	.102
D	.144	.106	.084	.076
U	.184	.141	.116	.103
V	.196	.156	.130	.114
W	.188	.158	.129	.112
X	.180	.136	.114	.098
Y	.166	.116	.094	.082
Z	.160	.120	.103	.088
AA	.190	.154	.128	.110
AB	.183	.142	.118	.102
AC	.156	.115	.094	.081
AD	.176	.156	.118	.092
AE	.192	.144	.114	.102

APPENDIX A

GENERAL FORM OF THE RESULTS OBTAINED

From Chapter IV it must be remembered that a company *i* generated for the study of an investment situation is defined by:

1. An initial capital.
2. An interest rate i_δ.
3. H + 1 sets of investment proposals SIP(T) for times $t = 0,1,2,\ldots,H$, generated by random sampling in the distributions that define the investment situation.

Each investment situation had eight such companies; for each of them, several simulations of a decision process were made, using tentative average marginal growth rates m, $m = i_\delta, i_\delta + .02,\ldots,.26$.

In the last period of the operation of each company i, the average growth rate of company i using a tentative marginal growth rate m, g(i,m), was computed, and the average of the values of g(i,m) was obtained on the eight companies. The average was named $\bar{g}(m)$.

Figure 14 presents in a graphical form the type of variation of $\bar{g}(m)$ with changes in m. The values of $\bar{g}(m)$ for the investment situations AE1, AE2, AE3 and AE4 are plotted as ordinates with values of m as abscissas. The smoothed curves were obtained through lineal regression on the values of $\bar{g}(m)$ for each investment situation as described on page 58. The values of $\bar{g}(m)$ are represented in Table 12.

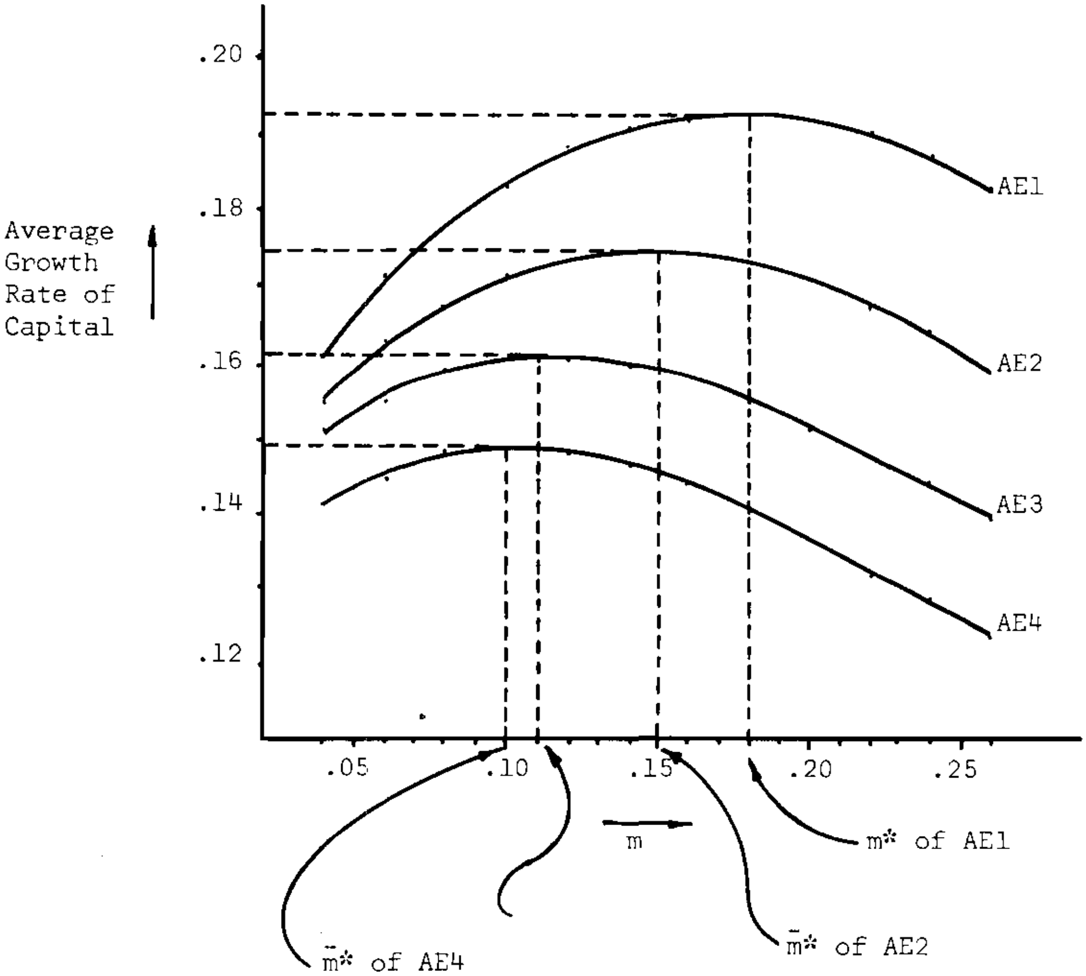


Figure 14. Variation of the Average Growth Rate of Capital at a Horizon Date with Changes in m

Table 12. Values of Average Growth Rate of Capital for Different Values of m for the Class of Investment Situations AE

Value of m	Investment Situation AE1	AE2	AE3	AE4
.04	.160	.154	.151	.141
.06	.172	.163	.153	.143
.08	.178	.168	.160	.149
.10	.183	.172	.161	.149
.12	.189	.174	.161	.148
.14	.191	.175	.160	.146
.16	.191	.175	.158	.143
.18	.193	.174	.155	.141
.20	.192	.172	.151	.137
.22	.191	.167	.148	.132
.24	.188	.166	.145	.129
.26	.183	.159	.139	.123

The highest point of the smoothed curves was obtained, and the value of the abscissa of such a point was rounded to the nearest one hundredth. These values were reported as the $\bar{m}^*$ for the investment situations:

$\bar{m}^*$ for AE1 = .18

$\bar{m}^*$ for AE2 = .15

$\bar{m}^*$ for AE3 = .11

$\bar{m}^*$ for AE4 = .10

APPENDIX B

Table 13. Values of $\bar{m}^*$ for the Investment Situations Studied

Class of Investment Situations	Initial Capital			
	10^5	2×10^5	3×10^5	4×10^5
A	.17	.12	.09	.08
B	.20	.17	.17	.14
C	.18	.15	.12	.11
D	.15	.10	.09	.06
E	.17	.12	.10	.08
F	.19	.17	.15	.13
G	.17	.15	.12	.10
H	.14	.10	.08	.07
I	.16	.15	.14	.14
J	.20	.18	.15	.14
K	.18	.14	.14	.14
L	.13	.12	.13	.13
M	.16	.13	.10	.08
N	.20	.15	.13	.12
O	.18	.15	.13	.11
P	.13	.09	.08	.07
Q	.18	.17	.14	.13
R	.19	.17	.13	.11
S	.18	.14	.12	.10
T	.17	.13	.11	.09
U	.19	.15	.12	.10
V	.19	.15	.13	.10
W	.18	.14	.13	.11
X	.19	.14	.12	.09
Y	.17	.12	.09	.07
Z	.18	.11	.10	.08
AA	.19	.16	.12	.11
AB	.18	.14	.12	.10
AC	.17	.12	.09	.07
AD	.17	.13	.10	.09
AE	.18	.15	.11	.10
AF	.18	.15	.13	.14
AG	.17	.15	.15	.14
AH	.18	.14	.12	.13
AI	.15	.12	.14	.15

Table 14. Values of $\bar{g}(\bar{m}^*)$ Found for the Investment Situations Studied

Class of Investment Situations	Initial Capital			
	10^5	2×10^5	3×10^5	4×10^5
A	.193	.170	.153	.140
B	.210	.192	.181	.171
C	.202	.184	.169	.156
D	.185	.158	.140	.127
E	.194	.172	.155	.142
F	.199	.187	.176	.167
G	.204	.184	.168	.155
H	.186	.157	.140	.126
I	.200	.178	.161	.150
J	.206	.196	.185	.176
K	.209	.190	.175	.163
L	.189	.164	.149	.139
M	.190	.168	.152	.140
N	.200	.185	.169	.159
O	.199	.185	.170	.158
P	.180	.154	.136	.123
Q	.207	.192	.177	.165
R	.196	.180	.167	.155
S	.203	.183	.166	.153
T	.196	.176	.159	.146
U	.211	.191	.174	.161
V	.194	.179	.167	.155
W	.195	.174	.160	.149
X	.199	.180	.164	.152
Y	.192	.165	.147	.133
Z	.196	.172	.154	.141
AA	.199	.182	.167	.155
AB	.194	.178	.162	.151
AC	.192	.164	.147	.133
AD	.196	.171	.155	.143
AE	.193	.175	.161	.149
AF	.202	.185	.170	.158
AG	.207	.189	.175	.163
AH	.201	.178	.163	.152
AI	.197	.170	.153	.143

LITERATURE CITED

1. R. V. Oakford and G. J. Thuesen, "The Maximum Prospective Value Criterion," *The Engineering Economist*, Vol. XIII, No. 3, pp. 141-4 (1968).

2. G. J. Thuesen, *Decision Techniques for Capital Budgeting Problems*, Ph.D. Thesis, Department of Industrial Engineering, Stanford University (1967).

3. R. V. Oakford and G. J. Thuesen, "The Effectiveness of the Maximum Prospective Value Criterion for Capital Budgeting Decisions," *Proceedings of the 19th Annual Institute Conference and Convention, American Institute of Industrial Engineers*, May, 1968.

4. R. H. Bernhard, "Discount Methods for Expenditure Evaluation: A Clarificatin of Their Assumptions," *Journal of Industrial Engineering*, Vol. XIII, No. 1, p. 19 (1962).

5. A. C. Williams and J. I. Nassar, "Financial Measurement of Capital Investments," *Management Science*, Vol. XII, No. 11, pp. 851-64 (1966).

6. H. M. Weingartner, *Mathematical Programming and the Analysis of Capital Budgeting Problems*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey (1963).

7. D. F. Istvan, *Capital-Expenditure Decisions, How They are Made in Large Corporations*, Indiana Business Report No. 33, Bureau of Business Research, Graduate School of Business. Indiana University (1961).

8. G. W. Smith, *Engineering Economy: Analysis of Capital Expenditures*, The Iowa State University Press, Ames, Iowa, pp. 467-98 (1968).

9. J. Dean, *Capital Budgeting*, Columbia University Press, New York, N. Y. (1951).

10. G. A. Fleischer, "Two Major Issues Associated with the Rate of Return Method for Capital Allocation: The 'Ranking Error' and 'Preliminary Selection'," *Journal of Industrial Engineering*, Vol. XVII, No. 4 (1966).

11. D. Teichroew, A. A., Robichek and M. Montalbano, "An Analysis of Criteria for Investment and Financing Decisions," *Management Science*, Vol. XII, No. 3, pp. 151-79 (1965).

12. E. L. Grant and W. G. Ireson, *Principles of Engineering Economy,* The Ronald Press Co., New York, N. Y. (1964).

13. J. Lorie and L. J. Savage, "The Problems in Rationing Capital," *Journal of Business,* Vol. XXVIII, No. 4, pp. 229-239 (1955).

14. E. F. Brigham and K. V. Smith, "Cost of Capital to the Small Firm," *The Engineering Economist,* Vol. XIII, No. 1, pp. 1-26 (1967).